Why Spreadsheets are Inadequate for Uncertainty Analysis

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Abstract

The importance of estimating and reporting measurement uncertainty has greatly increased over the past few years. As a result, many companies and organizations have developed, or are seriously considering the development of, spreadsheet applications or templates to address this need. This paper discusses key questions and concerns regarding the development of uncertainty analysis worksheets or custom add-in programs for Excel and Lotus spreadsheet applications.

Introduction

Testing and calibration standards such as ANSI/ISO/IEC 17025 [1] have elevated the importance of uncertainty analysis for achieving laboratory accreditation. ISO/TAG4/WG3 (the GUM) and ANSI/NCSL Z540-2-1997 [2] (the U.S. version of the GUM) provide general rules and guidelines for analyzing and communicating measurement uncertainty. However, implementing these guidelines into an effective uncertainty analysis tool requires a strong background in the necessary mathematical and statistical concepts.

Until about ten years ago, off-the-shelf uncertainty analysis programs were not available [3]. Prior to that time, spreadsheet applications were the best tools available for developing uncertainty estimates and budgets. As a developer and marketer of uncertainty analysis tools, Integrated Sciences Group (ISG) has considerable experience using spreadsheet programs to evaluate uncertainty for various measurement scenarios.

Our experience has shown that conducting a realistic measurement uncertainty analysis using spreadsheets can be labor intensive, often requiring the development of macros and other subroutines. In our evaluation of spreadsheet analyses developed by others, we have found many of them to be either overly simplistic or patched-together using inappropriate methods and techniques. Consequently, the analysis results are often meaningless and misleading.

Practical Considerations

One main advantage of spreadsheet programs, such as Excel or Lotus, is that most technical personnel routinely use them. A second advantage is that the uncertainty analysis developer has full control over the spreadsheet content, layout, equations and algorithms used. Unfortunately, there are also several significant disadvantages of using spreadsheet programs that are discussed herein. These disadvantages are the primary reasons why off-the-shelf uncertainty analysis programs are not typically designed as spreadsheet add-in programs.

Validity of Uncertainty Estimates

In any given measurement scenario, there are potential sources of error. These measurement process errors are the basic elements of uncertainty analysis. Once these fundamental error sources have been identified, we can begin to develop uncertainty estimates. The errors most often encountered in making measurements include, but are not limited to

- Measurement bias – the bias in the measuring device and/or the quantity being measured.
- Random or repeatability error – the error associated with repeat measurements.
- Resolution error – the error resulting from the finite resolution of the measuring device and/or the quantity being measured.
- Operator bias – the error introduced by the person making the measurements.
- Environmental factors error – the error introduced by variations in environmental conditions or by correcting for environmental conditions.

- Digital sampling error – the error introduced by digitizing an analog signal.
- Computation error – the error due to round-off or computer truncation, numerical interpolation, using empirically determined equations, etc.

Our lack of knowledge about the sign and magnitude of measurement error is called measurement uncertainty. Fortunately, measurement errors can be characterized by statistical distributions [4]. That is, errors can be described in such a way that their sign and magnitude have some definable probability of occurrence.

Error distributions include, but are not limited to normal, lognormal, uniform (rectangular), triangular, quadratic, cosine, exponential, u-shaped, trapezoidal and student's t. Each distribution is characterized by a set of statistics that include the mean, or mode, and the standard deviation. The uncertainty associated with a measurement process error is simply the standard deviation of the error distribution [3-5].

An uncertainty analysis application should assist the user in identifying error sources and selecting the appropriate error distributions. This, in part, can be achieved via templates and screens, as will be discussed further in the User Interface section. In addition, the application must be able to apply information provided by the user to compute the standard deviation of the appropriate distribution. Spreadsheet programs have a few built-in distribution functions, such as normal, lognormal and student's t. However, additional macros and subroutines must be developed to handle other distributions.

User Interface

Because most users will not have advanced training in uncertainty analysis methods, considerable effort is required to ensure that the design, layout and organization of the spreadsheets and associated screens provide sufficient technical guidance. The spreadsheet templates must query the user, via interactive means, to obtain the technical information needed to estimate uncertainties for various measurement error sources. This can be achieved by developing spreadsheet macros using, for example, the Visual Basic for Applications (VBA) programming language.

Unfortunately, it is not difficult for someone with a moderate familiarity with spreadsheet programs to access and modify many of these "behind-the-scenes" macros by simply copying the spreadsheet template(s). The primary reason for this is that spreadsheet programs like Excel or Lotus are specifically designed to provide easy access to the full functionality of the program. Consequently, employing password protections and hiding cells cannot completely prevent such access without significantly degrading overall user interaction.

Error Trapping of Data Input

An uncertainty analysis tool should always include error traps to ensure that realistic information and data are entered in the appropriate fields and cells. Error trapping is more difficult with Excel or Lotus spreadsheet programs because the cells in which data are entered cannot be completely secured.

Validation of Mathematical and Statistical Methods

It is important that any uncertainty analysis application or tool incorporates proper mathematical and statistical methods for estimating and combining measurement process uncertainties. This requires that the application developer have a sufficient technical understanding of uncertainty analysis concepts and principles.

Full implementation of the uncertainty analysis methods and procedures outlined in the GUM requires considerable programming effort, including the development of algorithms for computing partial derivatives of multivariate measurement equations. The resulting robust uncertainty analysis tool quickly becomes a full-fledged software application instead of a simple spreadsheet template.

If numerical partial derivative algorithms are not designed into the spreadsheet application, then considerable error can be introduced through manual differentiation or through the omission of necessary sensitivity coefficients. This is a major concern when developing and using spreadsheet templates to conduct uncertainty analyses. An example is included in the Appendix to illustrate the inherent weaknesses in many spreadsheet analyses.

Additionally, it is unwise to assume that MS Excel provides validated mathematical and statistical functions. Over the years our company has identified several instances where the MS Excel statistical functions provide insufficient precision for uncertainty analysis calculations. Some of the functions are not defined properly, producing incorrect results. Kurtosis, a measure of the peakedness of the distribution of a sample of data, is an obvious example.

Maintenance Issues

Maintenance of an uncertainty analysis application is an ongoing task that requires a long-term commitment of the developer. As previously indicated, it can be especially difficult to ensure the integrity of spreadsheet templates after they have been widely distributed. In addition, Microsoft Excel workbooks or add-in programs are particularly vulnerable to macro viruses written by individuals intent on destroying or corrupting data.

Simply opening an infected workbook can activate the virus. At present, Microsoft Excel does not have the capability to scan for and remove macro viruses. Instead, a warning message is displayed whenever a worksheet containing macros is opened. Therefore, sharing of spreadsheet templates can exacerbate the risk of spreading viruses.

Technical Support

Technical support is especially important when using a specialized analysis application or spreadsheet template. These analysis tools should include comprehensive on-screen Help features, a user manual, and options for contacting technical support personnel. The on-screen Help function should have complete index and search capabilities of all topics to facilitate use. Unfortunately, index and search capabilities of external Help files do not always function properly when launched from within spreadsheet programs. For example, the Excel Help index and search functions typically supersede those of external Help files.

Conclusions

Developing and using simplified spreadsheet templates can provide an unrealistic assessment of measurement uncertainty. Conversely, developing rigorous analysis tools requires considerable programming effort and technical expertise in the requisite mathematical and statistical methods and concepts.

When designing an uncertainty analysis tool within an Excel or Lotus spreadsheet program, it is difficult to ensure the integrity of templates or macros after they have been widely distributed. In addition, Excel workbooks and templates are particularly vulnerable to macro viruses, as are other Microsoft Office applications.

Through the development of uncertainty analysis applications over the past several years, ISG has helped many of our customers make the transition from labor intensive, statistically simplified, and patched-together spreadsheet analyses. This transition away from spreadsheet analyses to a more robust uncertainty analysis program has proven, time and time again, to be a better and more cost-effective solution.

References

[1] General Requirements for the Competence of Testing and Calibration Laboratories, ANSI/ISO/IEC 17025:2000.

[2] U.S. *Guide to the Expression of Uncertainty in Measurement*, ANSI/NSCL-Z540-2-1997.

[3] Castrup, S.: "A Comprehensive Comparison of Uncertainty Analysis Tools," presented at the 2004 Measurement Science Conference, January 15-17, 2004 in Anaheim, CA.

[4] Castrup, H.: "Distributions for Uncertainty Analysis," presented at the 2001 International Dimensional Workshop, May 2001 in Knoxville, TN.

[5] Castrup, H.: "Estimating Bias Uncertainty," presented at the 2001 NCSLI Workshop and Symposium, July 2001, Washington, DC.

[6] Panko, R.: "What We Know about Spreadsheet Errors," *Journal of End User Computing*, Vol 10, No 2., 1998.

[7] Microsoft® Excel 2003, Copyright© 1985-2003 Microsoft Corporation.

[8] UncertaintyAnalyzer 3.0, Copyright© 2006-2007, Integrated Sciences Group.

Appendix – Load Cell Analysis

A load cell analysis example is included herein to illustrate the difficulties associated with the correct implementation of uncertainty analysis methods into spreadsheet templates. The uncertainty in the load cell output voltage is estimated using both a system model analysis approach and a more general multivariate analysis method. Results obtained from Excel [6] spreadsheet analyses are compared to similar analyses using ISG's UncertaintyAnalyzer [7] application. In addition, these results are compared to results obtained via an incorrect, simplified root-sum-square (RSS) analysis method to illustrate a major weakness often encountered in spreadsheet analyses.

Measurement Process Overview

In this example, a load cell is calibrated using a weight standard, as illustrated in Figure 1. The calibration weight is extended from the load cell via a monofilament line and DC voltage output from the amplifier/signal conditioner is measured with a digital multimeter. Repeat measurements of DC voltage are obtained by adding and removing the calibration weight.

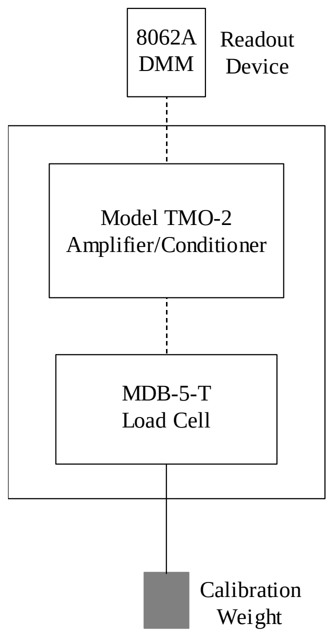


Figure 1 - Load Cell Calibration Setup

The purpose of this analysis is to estimate and report the total uncertainty in the average DC voltage obtained via the load cell calibration process. In this analysis, error in the mass of the calibration weight, errors intrinsic to the measurement equipment used, and other process errors are considered. A list of applicable error sources is given below.

- Bias in the value of the calibration weight
- Errors associated with the MDB-5-T Load Cell
- Errors associated with the Model TMO-2 Amplifier
- Errors associated with the 8062A Digital Multimeter
- Error associated with the repeat measurements taken

Uncertainty Analysis Procedure

For the load cell system analysis, we need to define the mathematical relationship between the quantity being investigated and its component variables. In this case, measurement is made through a linear sequence of stages as shown in Figure 2.

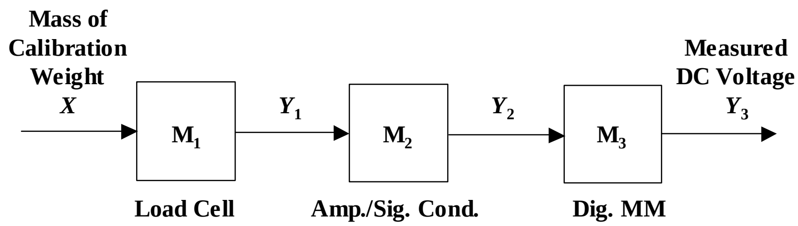


Figure 2 Block Diagram of Load Cell Calibration System

The output, *Y*, from any given module of the system may comprise the input of another module or modules. Since each module's output carries with it an element of uncertainty, this means that this uncertainty may be present at the input of a subsequent module.

As one would expect, system uncertainty analysis follows a structured procedure. Since a detailed block diagram has been established, we can develop the equations that relate the inputs and outputs for each module. The basic approach is to clearly describe the physical processes and identify sources of error that can affect the error in measured value.

Load Cell Module (M_1)

The first module consists of an MDB-5-T load cell manufactured by Transducer Techniques, Inc. This load cell is a passive sensor that requires an external voltage source and has a rated output of 2 mV/V nominal for loads up to 5 lbs. Therefore, the nominal sensitivity of the load cell is 0.4 mV/V/lb and the basic transfer function is

$$LC_{Out} = W \times S \times V_{ex} \quad (1)$$

where

LC_{Out} = Load cell output, mV
W = Applied load or weight
S = Load cell sensitivity, mV/V/lb
V_{ex} = Excitation voltage, V

For this module, we need to consider the following error sources:

- Bias in the value of the calibration weight
- Excitation voltage error
- Load cell error

Manufacturer's published specifications for the load cell[1] are listed in Table 1.

Table 1 MDB-5-T Load Cell Specifications

Specification	Value	Units
Maximum Applied Load	5	lbs
Rated Output (R.O.)	2	mV/V
Nonlinearity	0.05% of R.O.	mV/V
Hysteresis	0.05% of R.O.	mV/V
Noise (Nonrepeatability)	0.05% of R.O.	mV/V
Zero Balance	1.0% of R.O.	mV/V
Compensated Temp. Range	60 to 160	°F
Temperature Effect on Output	0.005% of Load/°F	lb/°F
Temperature Effect on Zero	0.005% of R.O./°F	mV/V/°F
Recommended Excitation Voltage	10	VDC

Therefore, we need to consider the following sources of load cell error:

[1] Specifications obtained from www.ttloadcells.com/mdb-load-cell.cfm

- Nonlinearity
- Hysteresis
- Noise
- Zero balance
- Temperature effect on output
- Temperature effect on zero

When developing an equation for the load cell module, we must consider what impact the error sources will have on the output. We will briefly discuss each of the error sources listed above and decide how they should be accounted for in the load cell output equation.

Calibration Weight

The nominal value of the calibration weight is stated to be 3 lb with error limits of ± 0.003 lb. In this analysis, we interpret these limits to represent a 99 % confidence interval. We also assume that the errors contained within these limits follow a normal distribution.

Excitation Voltage

Since the MDB-5-T load cell is a passive sensor, it requires an external power supply. The TMO-2 Amplifier provides a regulated 8 VDC excitation power supply with ± 0.25 V error limits. In this analysis, we interpret the excitation voltage error limits to be a 95% confidence interval.

Nonlinearity.

Nonlinearity is a measure of the deviation of the actual input-to-output performance of the device from an ideal linear relationship. Nonlinearity error is fixed at any given input, but varies with magnitude and sign over a range of inputs. Therefore, it is considered to be a random error that is normally distributed. In this analysis, we will interpret the manufacturer specification of
± 0.05% of the rated output to be a 95% confidence interval.

Hysteresis

Hysteresis indicates that the output of the device is dependent upon the direction and magnitude by which the input is changed. At any input value, hysteresis can be expressed as the difference between the ascending and descending outputs. Hysteresis error is fixed at any given input, but can vary with magnitude and sign over a range of inputs. Therefore, it is considered to be a random error that is normally distributed. In this analysis, we will interpret the manufacturer specification of ± 0.05% of the rated output to be a 95% confidence interval.

Noise

Nonrepeatability or random error intrinsic to the device, which causes the output to vary from observation to observation for a constant input is usually specified as noise. This error source varies with magnitude and sign over a range of inputs and is normally distributed. In this analysis, we will interpret the manufacturer specification of ± 0.05% of the rated output to be a 95% confidence interval.

Zero Balance

Zero balance refers to the zero offset that occurs if the device exhibits a non-zero output for a zero input. Although zero offset error can be reduced by adjustment, there is no way to completely eliminate it because we do not know the true value of the offset. In this analysis, we will interpret the manufacturer specification of ± 1% of the rated output to be a 95% confidence interval for a normally distributed error.

Temperature Effects

Temperature can affect both the offset and sensitivity of a device. To establish these effects, the device is typically tested at several temperatures within its operating range and the effects on zero and sensitivity or output are observed.

The temperature effect on output of 0.005% load/°F specified by the manufacturer is equivalent to 0.00015 lb/°F for an applied load of 3 lbs. The temperature effect on zero specification of 0.005% R.O./°F and the temperature effect on output are interpreted to be a 95% confidence interval for normally distributed errors.

For this analysis, we will use a temperature range of 10 °F with error limits of ± 2 °F with an associated 99% confidence level. The temperature measurement error is also assumed to be normally distributed.

Load Cell Output Equation

In developing the equation to compute the load cell output as a function of the input load (i.e., calibration weight), we need to first assign a consistent naming convention for the relevant error source and related parameters. The parameters used in the load cell output equation are listed in Table 2. The output equation for the load cell module is expressed in equation (2).

$$LC_{Out} = ((W_C + TE_{out} \times TR_{°F}) \times S + NL + Hys + NS + ZO + TE_{Zero} \times TR_{°F}) \times V_{ex} \quad (2)$$

Table 2 Parameters used in Load Cell Module Equation

Parameter Name	Description	Nominal or Mean Value	Error Limits	Percent Confidence
W_C	Calibration Weight or Load	3 (lb)	± 0.003 (lb)	99
S	Load Cell Sensitivity	0.4 (mV/V/lb)		
NL	Nonlinearity	0 (mV/V)	± 0.05% R.O. (mV/V)	95
Hys	Hysteresis	0 (mV/V)	± 0.05% R.O. (mV/V)	95
NS	Nonrepeatability	0 (mV/V)	± 0.05% R.O. (mV/V)	95
ZO	Zero Balance	0 (mV/V)	± 1% R.O. (mV/V)	95
$TR_{°F}$	Temperature Range	10 (°F)	± 2.0 (°F)	99
TE_{Out}	Temp Effect on Output	0 (lb/°F)	± 0.005% Load/°F (lb/°F)	95
TE_{Zero}	Temp Effect on Zero	0 (mV/V/°F)	± 0.005% R.O./°F (mV/V/°F)	95
V_{ex}	Applied Excitation Voltage	8 (V)	± 0.25 (V)	95

Amplifier/Signal Conditioner Module (M_2)

The TMO-2 Amplifier, manufactured by Transducer Techniques Inc., amplifies the load cell output from a mV to V. The nominal amplifier gain is the ratio of the maximum amplifier output to the maximum load cell output. The basic transfer function for this module is

$$Amp_{Out} = LC_{Out} \times G \quad (3)$$

where

Amp_{Out} = Amplifier Output, V
G = Amplifier Gain, V/mV

For this module, we need to consider the following error sources:

- Load cell error
- Amplifier error

Manufacturer's published specifications for the amplifier[2] are listed in Table 3. For a recommended applied excitation voltage of 10 VDC, the MDB-5-T load cell has a maximum rated output of 20 mV. Therefore, the TMO-2 amplifier has a nominal gain of 10V/20 mV or 0.5 V/mV.

Table 3 TMO-2 Amplifier Specifications

Specification	Value	Units
Maximum Output Voltage	10	V
Gain (nominal)	0.5	V/mV
Nonlinearity	0.01%	mV
Accuracy	0.05% of Full Scale	mV
Noise and Ripple	< 3	mV
Balance Stability	0.2%	mV

[2] Specifications obtained from www.ttloadcells.com/TMO-2.cfm

Specification	Value	Units
Gain Stability	0.01%	mV
Temperature Coefficient	0.02% of F.S./°C	mV/°C

Given the above specifications, we need to consider the following sources of amplifier error:

- Gain accuracy
- Gain stability (or Instability)
- Nonlinearity
- Noise
- Balance stability
- Temperature coefficient

Gain Accuracy
Gain is the ratio of the amplifier output signal voltage to the input signal voltage. In this case, the TMO-2 amplifier has a nominal gain of 10V/20 mV or 0.5 V/mV. In this analysis, we will interpret the manufacturer specified accuracy of ± 0.05% of full scale to be a 95% confidence interval for normally distributed errors.

Gain Stability
If the amplifier voltage gain is represented by G_V, its input resistance by R and its feedback resistance by R_f, then oscillations are possible when

$$\frac{RG_V}{R+R_f} = \pi \; .$$

These oscillations appear as an instability in the amplifier gain. In this analysis we will interpret the manufacturer specification of 0.01% to be ± 0.01% of full scale. We will assume these limits represent a 95% confidence interval for normally distributed errors.

Nonlinearity
As with the load cell module, actual amplifier response may depart from the ideal or assumed output versus input curve. Nonlinearity errors are point-by-point differences in actual versus expected response over the range of input signal levels. In this analysis, we will interpret the manufacturer specification of 0.01% to be ± 0.01% of full scale and represent a 95% confidence interval.

Noise
Noise generated within the amplifier that enters the signal path causes errors in the amplifier output. Since noise is directly related to gain, manufacturers usually specify noise error in absolute units of Volts RMS or Volts peak-to-peak. In this analysis, we will interpret the manufacturer specification of 3 mV peak-to-peak to be a 99% confidence interval for normally distributed errors.

Balance Stability
Balance stability, or instability, refers to a non-zero amplifier output exhibited for a zero input. Although balance instability can be reduced by adjustment, there is no way to completely eliminate it because we do not know the true value of the zero offset. In this analysis, we will interpret the manufacturer specification of ± 0.2% to be ± 0.2% of full scale and that this reflects a 95% confidence interval for normally distributed errors.

Temperature coefficient
Both the balance (or zero) and gain are affected by temperature. Manufacturers generally state this as a temperature coefficient (or Tempco) in terms of percent change or full scale per degree. In this analysis, we will interpret the manufacturer specification of ± 0.02% of full scale/°C to be a 95% confidence interval for normally distributed errors.

To quantify the effect of temperature, however, we must establish the expected temperature change and use this with the temperature coefficient to compute expected variations. As with the load cell module, we will estimate the impact of temperature correction error using a temperature range of 10 °F (5.6 °C) with measurement error limits of ± 1.1 °C with an associated confidence level of 99% for normally distributed errors.

Amplifier Output Equation

The output equation for the amplifier module is expressed below. Naming conventions and error limits for the various parameters are listed in Table 4. The amplifier output equation is expressed in equation (4).

$$Amp_{Out} = LC_{Out} \times G + G_{Acc} + G_S + G_{NL} + G_{NS} + B_{St} + TC \times TR_{^\circ C} \quad (4)$$

Table 4 Parameters used in Amplifier Module Equation

Parameter Name	Description	Nominal or Mean Value	Error Limits	Percent Confidence
LC_{Out}	Amplifier Input			
G	Gain	0.5 (V/mV)		
G_{Acc}	Gain Accuracy	0 (V)	± 0.05% FS (mV)	95
G_S	Gain Stability	0 (V)	± 0.01% FS (mV)	95
G_{NL}	Nonlinearity	0 (V)	± 0.01% FS (mV)	95
G_{NS}	Noise	0 (V)	± 3 (mV)	99
B_{St}	Balance Stability	0 (V)	± 0.2% FS (mV)	95
TC	Temperature Coefficient	0 (V/°C)	± 0.02% FS/°C (mV/°C)	95
$TR_{^\circ C}$	Temperature Range	5.6(°C)	± 1.1 (°C)	99

Digital Multimeter Module (M_3)

The 8602A digital multimeter, manufactured by Fluke, converts the analog output signal from the amplifier module to a digital signal and displays it on a readout device. The basic transfer function for this module is expressed in equation (5).

$$DMM_{Out} = Amp_{Out} \quad (5)$$

where

DMM_{Out} = Digital multimeter output, V

Manufacturer's published specifications for the DC voltage function of the digital multimeter[3] are listed in Table 5. In this module, key error sources include:

- DC voltmeter accuracy
- DC voltmeter digital resolution
- Repeat measurements error

Table 5 8062A DC Voltage Specifications

Specification	Value	Units
200 mV Range Resolution	0.01	mV
200 mV Range Accuracy	0.05% of Reading + 2 digits	mV
2 V Range Resolution	0.1	mV
2 V Range Accuracy	0.05% of Reading + 2 digits	mV
20 V Range Resolution	1	mV
20 V Range Accuracy	0.07% of Reading + 2 digits	mV

DC Voltage Accuracy.

The overall accuracy of the DC Voltage reading for a 20 V range is specified as ± (0.07% of reading + 2 digits). In this analysis, we will interpret these specifications to be a 95% confidence interval for normally distributed errors.

[3] Specifications from 8062A Instruction Manual downloaded from www.fluke.com

Digital Resolution.
The digital resolution for the 20 V DC range is specified as 1 mV. Since this is a digital display, the resolution error is uniformly distributed. Therefore, we will interpret the 100% confidence limits to be
± 0.5 mV.

Repeatability.
Random error resulting from repeat measurements can result from various physical phenomena such as temperature variation or the act of removing and re-suspending the calibration weight multiple times. Uncertainty due to repeatability error will be estimated using the data listed in Table 6.

Table 6 DC Voltage Readings

Repeat Measurement	VDC Measured (V)	VDC Deviation from Nominal (mV)
1	4.856	56
2	4.861	61
3	4.860	60

Digital Multimeter Output Equation.
The output equation for the digital multimeter module is expressed in equation (6).

$$DMM_{Out} = Amp_{Out} + DMM_{Acc} + DMM_{res} + V_{ran} \quad (6)$$

The relevant equation parameters and error limits are listed in Table 7.

Table 7 Parameters used in Multimeter Module Equation

Parameter Name	Description	Nominal or Mean Value	Error Limits	Percent Confidence
Amp_{Out}	DMM Input	4.80 (V)		
DMM_{Acc}	DC Voltmeter Accuracy	0 (V)	± (0.07% Read + 2) (mV)	95
DMM_{res}	DC Voltmeter Digital Resolution	0 (V)	± 0.5 (mV)	100
V_{ran}	Repeatability Error	(V)		

System Output and Total Uncertainty
The individual module equations and parameter information can now be used to estimate the output from the Load Cell Calibration System and the associated total uncertainty. There are a couple of ways to do this:

1. Analyze the three system modules separately and account for error propagation from module input to module output sequentially. This is the preferred analysis approach for this measurement scenario.
2. Alternatively, a multivariate analysis approach can be conducted using an overall system equation, and any associated nested variables equations, to describe the load cell calibration system.

If done correctly, the computed total system output and associated uncertainty should be the same for both analysis methods.

A third, but incorrect, method would be to estimate the uncertainties for all the error sources and then simply combine them in root-sum-square (RSS). However, as outlined in references [2] and [3], the correct method for combining uncertainties from different error sources must take into account sensitivity coefficients. These coefficients are the partial derivatives of the module equations or the overall system equation with respect to the individual error sources. They determine the relative contributions of the uncertainties in individual error sources to overall uncertainty.

The two correct analysis methods will be described and compared using both the Microsoft Excel[4] and ISG's UncertaintyAnalyzer application.[5] The third incorrect analysis method will also be discussed to illustrate the problems encountered when a "simplified" RSS approach is used.

System Analysis Method
In the system analysis approach, each module is analyzed separately and the outputs and associated uncertainties for each module are propagated to subsequent modules. The appropriate module equations are repeated below for reference.

Load Cell Module
$LC_{Out} = ((W_C + TE_{out} \times TR_{°F}) \times S + NL + Hys + NS + ZO + TE_{Zero} \times TR_{°F}) \times V_{ex}$

Amplifier Module
$Amp_{Out} = LC_{Out} \times G + G_{Acc} + G_S + G_{NL} + G_{NS} + B_{St} + TC \times TR_{°C}$

Digital Multimeter Module
$DMM_{Out} = Amp_{Out} + DMM_{Acc} + DMM_{res} + V_{ran}$

When conducting this analysis via Excel spreadsheet, the partial derivative equations for each parameter and error source coefficient were determined offline. The partial derivative equations are listed below for reference.

Load Cell Module

The sensitivity coefficients, computed by taking the partial derivatives of the load cell output equation (2), are listed below.

$$c_{W_C} = \frac{\partial LC_{Out}}{\partial W_C} = S \times V_{ex} \qquad c_S = \frac{\partial LC_{Out}}{\partial S} = W_C \times V_{ex} \qquad c_{NL} = \frac{\partial LC_{Out}}{\partial NL} = V_{ex}$$

$$c_{Hys} = \frac{\partial LC_{Out}}{\partial Hys} = V_{ex} \qquad c_{NS} = \frac{\partial LC_{Out}}{\partial NS} = V_{ex} \qquad c_{ZO} = \frac{\partial LC_{Out}}{\partial ZO} = V_{ex}$$

$$c_{TE_{Out}} = \frac{\partial LC_{Out}}{\partial TE_{Out}} = TR_{°F} \times S \times V_{ex} \qquad c_{TE_{Zero}} = \frac{\partial LC_{Out}}{\partial TE_{Zero}} = TR_{°F} \times V_{ex}$$

$$c_{TR_{°F}} = \frac{\partial LC_{Out}}{\partial TR_{°F}} = (TE_{out} \times S + TE_{zero}) \times V_{ex}$$

$$c_{V_{ex}} = \frac{\partial LC_{Out}}{\partial V_{ex}} = \left(W_C + TE_{Out} \times TR_{°F}\right) \times S + NL + Hys + NS + ZO + TE_{Zero} \times TR_{°F}$$

It is important to note that the sensitivity coefficients are computed using the parameter nominal or mean values.

Amplifier Module

The partial derivative equations used to compute the sensitivity coefficients for the amplifier output equation (4) are listed below.

$$c_{LC_{Out}} = \frac{\partial Amp_{Out}}{\partial LC_{Out}} = G \qquad c_G = \frac{\partial Amp_{Out}}{\partial G} = LC_{Out} \qquad c_{G_{Acc}} = \frac{\partial Amp_{Out}}{\partial G_{Acc}} = 1$$

$$c_{G_S} = \frac{\partial Amp_{Out}}{\partial G_S} = 1 \qquad c_{G_{NL}} = \frac{\partial Amp_{Out}}{\partial G_{NL}} = 1 \qquad c_{G_{NS}} = \frac{\partial Amp_{Out}}{\partial G_{NS}} = 1$$

$$c_{B_{St}} = \frac{\partial Amp_{Out}}{\partial B_{St}} = 1 \qquad c_{TC} = \frac{\partial Amp_{Out}}{\partial TC} = TR_{°C} \qquad c_{TR_{°C}} = \frac{\partial Amp_{Out}}{\partial TR_{°C}} = TC$$

Digital Multimeter Module

The partial derivative equations used to compute the sensitivity coefficients for the digital multimeter output equation (6) are listed below.

$$c_{Amp_{Out}} = \frac{\partial DMM_{Out}}{\partial Amp_{Out}} = 1 \qquad c_{DMM_{Acc}} = \frac{\partial DMM_{Out}}{\partial DMM_{Acc}} = 1 \qquad c_{DMM_{res}} = \frac{\partial DMM_{Out}}{\partial DMM_{res}} = 1 \qquad c_{V_{ran}} = \frac{\partial DMM_{Out}}{\partial V_{ran}} = 1$$

The module equations, partial derivative equations and error source data were entered into an Excel spreadsheet. The inverse normal distribution function was also included to compute the coverage factors for specified confidence levels. The analysis results are summarized in Tables 8 through 10.

Table 8 Spreadsheet Analysis for Load Cell Module

					Parameter Value / Error Source Table								
Variable Name	Error Distribution	Error Limits	Units	% Conf.	Coverage Factor	Std Unc.	Units	Deg. Freedom	Sensitivity Coeff.	Component Unc.	Units	Nominal or Mean Value	Units
W_C	Normal	0.003	lb	99	2.5758	0.0012	lb	infinite	3.2	0.0037	mV	3	lb
TE_{out}	Normal	1.50E-04	lb/deg F	95	1.9600	0.0001	lb/deg F	infinite	32	0.0024	mV	0	lb/deg F
$TR_{°F}$	Normal	2	deg F	99	2.5758	0.7764	deg. F	infinite	0	0	mV	10	deg F
S				95					24			0.4	mV/V/lb
NL	Normal	0.001	mV/V	95	1.9600	0.0005	mV/V	infinite	8	0.0041	mV	0	mV
Hys	Normal	0.001	mV/V	95	1.9600	0.0005	mV/V	infinite	8	0.0041	mV	0	mV
NS	Normal	0.001	mV/V	95	1.9600	0.0005	mV/V	infinite	8	0.0041	mV	0	mV
ZO	Normal	0.02	mV/V	95	1.9600	0.0102	mV/V	infinite	8	0.0816	mV	0	mV
TE_{Zero}	Normal	0.0001	mV/V/deg F	95	1.9600	0.0001	mV/V/deg F	infinite	80	0.0041	mV	0	mV/deg F
V_{ex}	Normal	0.25	V	95	1.9600	0.1276	V	infinite	1.2	0.1531	mV	8	V
			Load Cell Output =		9.60	mV		**Uncertainty =**		0.1737	mV		

Table 9 Spreadsheet Analysis for Amplifier Module

					Parameter Value / Error Source Table								
Variable Name	Error Distribution	Error Limits	Units	% Conf.	Coverage Factor	Std Unc.	Units	Deg. Freedom	Sensitivity Coeff.	Component Unc.	Units	Nominal or Mean Value	Units
LC_{Out}						0.1737	mV	infinite	500	86.8610	mV	9.60	mV
G				95					9.6		mV	0.5	V/mV
G_{Acc}	Normal	5	mV	95	1.9600	2.551	mV	infinite	1	2.5511	mV	0	V
G_S	Normal	1	mV	95	1.9600	0.510	mV	infinite	1	0.5102	mV	0	V
G_{NL}	Normal	1	mV	95	1.9600	0.510	mV	infinite	1	0.5102	mV	0	V
G_{NS}	Normal	3	mV	99	2.5758	1.165	mV	infinite	1	1.1647	mV	0	V
B_{St}	Normal	20	mV	95	1.9600	10.204	mV	infinite	1	10.2043	mV	0	V
TC	Normal	2	mV/deg C	95	1.9600	1.020	mV/deg C	infinite	5.6	5.7144	mV	0	V/deg C
$TR_{°C}$	Normal	1.1	deg. C	99	2.5758	0.427	deg. C	infinite	0			5.6	deg. C
			Amplifier Output =		4.800	V		**Uncertainty =**		87.7	mV or	0.0877	V

Table 10 Spreadsheet Analysis for Digital Multimeter Module

					Parameter Value / Error Source Table								
Variable Name	Error Distribution	Error Limits	Units	% Conf.	Coverage Factor	Std Unc.	Units	Deg. Freedom	Sensitivity Coeff.	Component Unc.	Units	Nominal or Mean Value	Units
Amp_{Out}						0.0877	V		1	0.0877	V	4.800	V
DMM_{Acc}	Normal	0.0087	V	95	1.9600	0.0044	V	infinite	1	0.0044	V	0	V
DMM_{res}	Uniform	0.0005	V	100	1.7321	0.0003	V	infinite	1	0.0003	V	0	V
V_{ran}						0.0015	V	infinite	1	0.0015	V	0.059	V
		Digital Multimeter Output =			4.859	V		**Uncertainty =**		0.0878	V or	87.8	mV

The overall output and total uncertainty for the Load Cell Calibration System is equal to the values computed for the last module in the series. Therefore, the overall output is 4.859 V with a total uncertainty of 87.8 mV.

The same analysis was conducted using UncertaintyAnalyzer by entering the module equations and associated information into the System Model screen. Partial derivatives were automatically computed, eliminating the time and potential error associated with doing this manually. The analysis results are summarized in the UncertaintyAnalyzer reports shown in Figures 3 through 6.

Uncertainty Analysis Report **System Module Analysis Results**

Load Cell Calibration System

06-Jul-2007

Module 1: Load Cell
Input Measurement Area: Mass
Output Measurement Area: DC Voltage

Analysis Results:

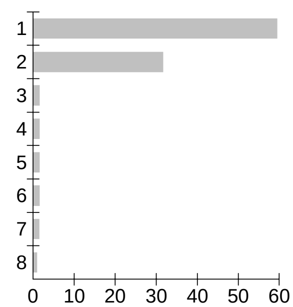
Name	Value	± Error Limits	% Confidence	Standard Uncertainty	Analysis Type	Deg Freedom	Sensitivity Coefficient	Component Uncertainty
TEOut	0	0.0001500	95.00	0.0000765	B	inf	32.0	0.0024490
TRdegF	10	2.0000	99.00	0.776		inf		
S	0.4		95.00				24.0	
NL	0	0.0010	95.00	0.00051	B	inf	8.0	0.00408
Hys	0	0.0010	95.00	0.00051	B	inf	8.0	0.00408
NS	0	0.0010	95.00	0.00051	B	inf	8.0	0.00408
ZO	0	0.0200	95.00	0.0102	B	inf	8.0	0.0816
TEZero	0	0.0001	95.00	0.000051	B	inf	80.0	0.004082
Vex	8	0.2500	95.00	0.1276	B	inf	1.20	0.1531

Module Analysis Summary:

Module Input: 3 lb
Input Uncertainty: 0.00384 lb
Degrees of Freedom: Infinite
Module Output: 9.60 mV
Output Uncertainty: 0.1737 mV
Distribution: Normal
Degrees of Freedom: Infinite
Analysis Category: Type B

Pareto Diagram:

Rank	Error Component	Type	Weight (%)
1	Vex	B	59.485
2	ZO	B	31.725
3	Hys	B	1.586
4	NS	B	1.586
5	NL	B	1.586
6	TEZero	B	1.586
7	Wc		1.492
8	TEOut	B	0.952



Module Output Equation:

LCOut = ((Wc + TEOut * TRdegF) * S + NL + Hys + NS + ZO + TEZero * TRdegF) * Vex

Figure 3 UncertaintyAnalyzer Summary Report for Load Cell Module

Uncertainty Analysis Report **System Module Analysis Results**

Load Cell Calibration System

06-Jul-2007

Module 2: Amplifier
Input Parameter: Load Cell
Input Measurement Area: DC Voltage
Output Measurement Area: DC Voltage

Analysis Results:

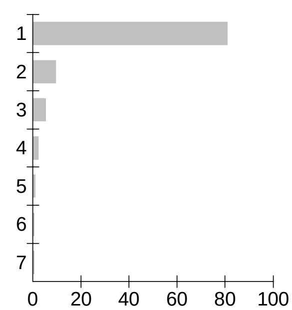
Name	Value	± Error Limits	% Con-fidence	Standard Uncertainty	Analysis Type	Deg Freedom	Sensitivity Coefficient	Component Uncertainty
G	0.5		95.00				9.60	
GAcc	0	0.0050	95.00	0.00255	B	inf	1.0	0.00255
GS	0	0.0010	95.00	0.00051	B	inf	1.0	0.00051
GNL	0	0.0010	95.00	0.00051	B	inf	1.0	0.00051
GNS	0	0.0030	99.00	0.001165	B	inf	1.0	0.001165
BSt	0	0.0200	95.00	0.0102	B	inf	1.0	0.0102
TC	0	0.0020	95.00	0.00102	B	inf	5.60	0.00571
TRdegC	5.6	1.1000	99.00	0.427		inf		

Module Analysis Summary:

Module Input: 9.60 mV
Input Uncertainty: 0.1737 mV
Degrees of Freedom: Infinite
Module Output: 4.80 V
Output Uncertainty: 87.7 mV
Distribution: Normal
Degrees of Freedom: Infinite
Analysis Category: Type B

Pareto Diagram:

Rank	Error Component	Type	Weight (%)
1	LCOut		80.787
2	Bst	B	9.492
3	TC	B	5.315
4	GAcc	B	2.373
5	GNS	B	1.083
6	GNL	B	0.475
7	GS	B	0.475



Module Output Equation:

AmpOut = LCOut * G + GAcc + GS + GNL + GNS + Bst + TC * TRdegC

Figure 4 UncertaintyAnalyzer Summary Report for Amplifier Module

Uncertainty Analysis Report **System Module Analysis Results**

Load Cell Calibration System

06-Jul-2007

Module 3: Digital Multimeter
Input Parameter: Amplifier
Input Measurement Area: DC Voltage
Output Measurement Area: DC Voltage

Analysis Results:

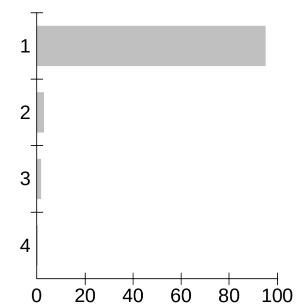
Name	Value	± Error Limits	% Con-fidence	Standard Uncertainty	Analysis Type	Deg Freedom	Sensitivity Coefficient	Component Uncertainty
DMMAcc	0	0.005360	95.00	0.002735	B	inf	1.0	0.002735
DMMres	0	0.0005000	100.00	0.0002887	B	inf	1.0	0.0002887
DMMran	0.0590	0.00657241	95.00	0.001528	A	2	1.0	0.001528

Module Analysis Summary:

Module Input: 4.80 V
Input Uncertainty: 87.7 mV
Degrees of Freedom: Infinite
Module Output: 4.86 V
Output Uncertainty: 87.8 mV
Distribution: Normal
Degrees of Freedom: Infinite
Analysis Category: Type A,B

Pareto Diagram:

Rank	Error Component	Type	Weight (%)
1	AmpOut		95.067
2	DMMAcc	B	2.964
3	Vran	A	1.656
4	DMMres	B	0.313



Module Output Equation:

DMMOut = AmpOut + DMMAcc + DMMres + Vran

Figure 5 UncertaintyAnalyzer Summary Report for Digital Multimeter Module

Uncertainty Analysis Report **System Analysis Results**

Load Cell Calibration System
06-Jul-2007

System Name: Load Cell Calibration System

Module Name	Input	Output	Uncertainty	Coefficient
Load Cell	3 lb	9.60 mV	0.1737 mV	0.0
Amplifier	9.60 mV	4.80 V	87.7 mV	0.0
Digital Multimeter	4.80 V	4.86 V	87.8 mV	1.000

System Analysis Summary:
System Input: 3 lb
System Input Uncertainty: 0.001 lb
Degrees of Freedom: Infinite
System Output: 4.86 V
System Output Uncertainty: 87.8 mV
Confidence Level: 95.00 %
Coverage Factor: 1.9590 mV
Degrees of Freedom: Infinite
Tolerance Limits: 172 mV

Figure 6 UncertaintyAnalyzer Summary Report for Load Cell Calibration System

As expected, the analysis results obtained via spreadsheet analysis and UncertaintyAnalyzer are essentially identical. The main difference is that it took 3 to 4 hours to perform the necessary off-line calculations, develop the spreadsheet analysis and double-check the calculations. In contrast, it took less than 30 minutes to enter the necessary system module equations and error source data into UncertaintyAnalyzer.

Multivariate Analysis Method
In the multivariate analysis approach, an overall equation is entered for the load cell calibration system, along with nested variables equations as shown below.

Overall Load Cell Calibration System Equation

$$System_{Out} = LC_{Out} \times G + G_{error} + DMM_{error} + V_{ran} \quad (7)$$

where

LC_{Out} = Load cell output, mV
G = Amplifier gain, V/mV
G_{error} = Amplifier error
DMM_{error} = Digital multimeter error
V_{ran} = Repeatability error

Load Cell Output Equation

$$LC_{Out} = ((W_C + TE_{out} \times TR_{°F}) \times S + NL + Hys + NS + ZO + TE_{Zero} \times TR_{°F}) \times V_{ex} \quad (8)$$

Amplifier Error Equation

$$G_{error} = G_{Acc} + G_S + G_{NL} + G_{NS} + B_{St} + TC * TR_{°C} \quad (9)$$

Digital Multimeter Error Equation

$$DMM_{Error} = DMM_{Acc} + DMM_{res} + V_{ran} \quad (10)$$

As was done for the system analysis, the partial derivatives equations for each error source coefficient were determined offline. It is important to note that, we are only interested in the sensitivity coefficients for the root variables or error sources, not the nested variables such as LC_{Out}, G_{error} or DMM_{Error}. The partial derivative equations are listed below for reference.

$$c_G = \frac{\partial System_{Out}}{\partial G} = LC_{Out} = \left(\left(W_C + TE_{Out} \times TR_{°\mathrm{F}}\right) \times S + NL + Hys + NS + ZO + TE_{Zero} \times TR_{°\mathrm{F}}\right) \times V_{ex}$$

$$c_{V_{ran}} = \frac{\partial System_{Out}}{\partial V_{ran}} = 1 \qquad c_{w_c} = \frac{\partial LC_{Out}}{\partial W_C} \times \frac{\partial System_{Out}}{\partial LC_{Out}} = S \times V_{ex} \times G$$

$$c_{TE_{Out}} = \frac{\partial LC_{Out}}{\partial TE_{Out}} \times \frac{\partial System_{Out}}{\partial LC_{Out}} = TR_{\circ\mathrm{F}} \times S \times V_{ex} \times G$$

$$c_{TR_{\circ\mathrm{F}}} = \frac{\partial LC_{Out}}{\partial TR_{\circ\mathrm{F}}} \times \frac{\partial System_{Out}}{\partial LC_{Out}} = (TE_{out} \times S + TE_{zero}) \times V_{ex} \times G$$

$$c_S = \frac{\partial LC_{Out}}{\partial S} \times \frac{\partial System_{Out}}{\partial LC_{Out}} = W_C \times V_{ex} \times G \qquad c_{NL} = \frac{\partial LC_{Out}}{\partial NL} \times \frac{\partial System_{Out}}{\partial LC_{Out}} = V_{ex} \times G$$

$$c_{Hys} = \frac{\partial LC_{Out}}{\partial Hys} \times \frac{\partial System_{Out}}{\partial LC_{Out}} = V_{ex} \times G \qquad c_{NS} = \frac{\partial LC_{Out}}{\partial NS} \times \frac{\partial System_{Out}}{\partial LC_{Out}} = V_{ex} \times G$$

$$c_{ZO} = \frac{\partial LC_{Out}}{\partial ZO} \times \frac{\partial System_{Out}}{\partial LC_{Out}} = V_{ex} \times G \qquad c_{TE_{Zero}} = \frac{\partial LC_{Out}}{\partial TE_{Zero}} \times \frac{\partial System_{Out}}{\partial LC_{Out}} = TR_{\circ\mathrm{F}} \times V_{ex} \times G$$

$$c_{V_{ex}} = \frac{\partial LC_{Out}}{\partial V_{ex}} \times \frac{\partial System_{Out}}{\partial LC_{Out}} = \left(\left(W_C + TE_{Out} \times TR_{\circ\mathrm{F}} \right) \times S + NL + Hys + NS + ZO + TE_{Zero} \times TR_{\circ\mathrm{F}} \right) \times G$$

$$c_{G_{Acc}} = \frac{\partial G_{error}}{\partial G_{Acc}} \times \frac{\partial System_{Out}}{\partial G_{error}} = 1 \qquad c_{G_S} = \frac{\partial G_{error}}{\partial G_S} \times \frac{\partial System_{Out}}{\partial G_{error}} = 1 \qquad c_{G_{NL}} = \frac{\partial G_{error}}{\partial G_{NL}} \times \frac{\partial System_{Out}}{\partial G_{error}} = 1$$

$$c_{G_{NS}} = \frac{\partial G_{error}}{\partial G_{NS}} \times \frac{\partial System_{Out}}{\partial G_{error}} = 1 \qquad c_{B_{St}} = \frac{\partial G_{error}}{\partial B_{St}} \times \frac{\partial System_{Out}}{\partial G_{error}} = 1$$

$$c_{TC} = \frac{\partial G_{error}}{\partial TC} \times \frac{\partial System_{Out}}{\partial G_{error}} = TR_{\circ\mathrm{C}} \qquad c_{TR_{\circ\mathrm{C}}} = \frac{\partial G_{error}}{\partial TR_{\circ\mathrm{C}}} \times \frac{\partial System_{Out}}{\partial G_{error}} = TC$$

$$c_{DMM_{Acc}} = \frac{\partial DMM_{Error}}{\partial DMM_{Acc}} \times \frac{\partial System_{Out}}{\partial DMM_{Error}} = 1 \qquad c_{DMM_{res}} = \frac{\partial DMM_{Error}}{\partial DMM_{res}} \times \frac{\partial System_{Out}}{\partial DMM_{Error}} = 1$$

The system output and nest variables equations, partial derivative equations and error source data were entered into an Excel spreadsheet. The inverse normal distribution function was also included to compute the coverage factors for specified confidence levels. The analysis results are summarized in Table 11.

The same multivariate analysis was conducted using UncertaintyAnalyzer by entering the system output and nested variables equations and error source data into the User Defined screen. Partial derivatives were automatically computed, eliminating the time and potential error associated with doing this manually. The results are summarized in the Multivariate Analysis summary report and Pareto diagram are shown in Figures 7 and 8, repectively.

Table 11 Spreadsheet Multivariate Analysis for Load Cell Calibration

					Parameter Value / Error Source Table								
Variable Name	Error Distribution	Error Limits	Units	% Conf.	Coverage Factor	Std Unc.	Units	Deg. Freedom	Sensitivity Coeff.	Component Unc.	Units	Nominal or Mean Value	Units
G				95					9.6			0.5	V/mV
Vran			mV			2.65	mV	2	1	0.00265	V	0.059	V
Wc	Normal	0.003	lb	99	2.5758	0.00116	lb	infinite	1.6	0.00186	V	3	lb
TEOut	Normal	0.00015	lb/deg F	95	1.9600	0.0000765	lb/deg F	infinite	16	0.001225	V	0	lb/deg F
TRdegF	Normal	2	deg F	99	2.5758	0.7764	deg. F	infinite				10	deg. F
S									12			0.4	mV/V/lb
NL	Normal	0.001	mV/V	95	1.9600	0.0005	mV	infinite	4	0.00204	V	0	mV
Hys	Normal	0.001	mV/V	95	1.9600	0.0005	mV	infinite	4	0.00204	V	0	mV
NS	Normal	0.001	mV/V	95	1.9600	0.0005	mV	infinite	4	0.00204	V	0	mV
ZO	Normal	0.02	mV/V	95	1.9600	0.0102	mV	infinite	4	0.0408	V	0	mV
TEZero	Normal	0.0001	mV/V/deg F	95	1.9600	0.0001	mV/deg F	infinite	40	0.00204	V	0	mV/deg F
Vex	Normal	0.25	V	95	1.9600	0.1276	V	infinite	0.600	0.076532132	V	8	V
GAcc	Normal	5	mV	95	1.9600	2.5511	mV	infinite	1	0.0026	V	0	V
GS	Normal	1	mV	95	1.9600	0.5102	mV	infinite	1	0.0005	V	0	V
GNL	Normal	1	mV	95	1.9600	0.5102	mV	infinite	1	0.0005	V	0	V
GNS	Normal	3	mV	99	2.5758	1.1647	mV	infinite	1	0.00116	V	0	V
BSt	Normal	20	mV	95	1.9600	10.2043	mV	infinite	1	0.0102	V	0	V
TC	Normal	2	mV/deg C	95	1.9600	1.0204	mV/deg C	infinite	5.6	0.0057	V	0	V/deg C
TRdegC	Normal	1.1	deg C	99	2.5758	0.4270	deg. C	infinite				5.6	deg. C
DMMAcc	Normal	5.36	mV	95	1.9600	2.73	mV	infinite	1	0.00273	V	0	V
DMMRes	Uniform	0.5	mV	100	1.7321	0.2887	mV	infinite	1	0.00029	V	0	V
				System Output =		4.859	V	**Total Uncertainty =**		0.0878	V or	87.8	mV

Uncertainty Analysis Report **Multivariate Uncertainty Analysis**

User Defined Analysis
06-Jul-2007

Measurand Data:
Parameter Name: Voltage Output from Load Cell Calibration System
Qualifier 1:
Qualifier 2:

Root Variables Analysis Results:

Variable Name	± Error Limits	% Confidence	Standard Uncertainty	Analysis Type	Deg. Freedom	Sensitivity Coefficient	Component Uncertainty	Adjusted Mean
G						9.60		0.5 V/mV
Vran			2.65 mV	A	2	1.0	0.00265 V	0.0590 V
Wc	0.003 lb	99.00	0.00116 lb	B	inf	1.60	0.001856 V	3 lb
TEOut	0.00015 lb/deg F	95.00	0.000077 lb/deg F	B	inf	16.0	0.001232 V	0 lb/deg F
TRdegF	2 deg F	99.00	0.78 deg F	B	inf			10 deg F
S						12.0		0.4 mV/V/lb
NL	0.001 mV/V	95.00	0.00051 mV/V	B	inf	4.0	0.00204 V	0 mV/V
Hys	0.001 mV/V	95.00	0.00051 mV/V	B	inf	4.0	0.00204 V	0 mV/V
NS	0.001 mV/V	95.00	0.00051 mV/V	B	inf	4.0	0.00204 V	0 mV/V
ZO	0.02 mV/V	95.00	0.0102 mV/V	B	inf	4.0	0.0408 V	0 mV/V
TEZero	0.0001 mV/V/deg F	95.00	0.000051 mV/V/deg F	B	inf	40.0	0.00204 V	0 mV/V/deg F
Vex	0.25 V	95.00	0.1276 V	B	inf	0.60	0.0766 V	8 V
GAcc	5 mV	95.00	2.55 mV	B	inf	1.0	0.00255 V	0 V
GS	1 mV	95.00	0.51 mV	B	inf	1.0	0.00051 V	0 V
GNL	1 mV	95.00	0.51 mV	B	inf	1.0	0.00051 V	0 V
GNS	3 mV	95.00	1.53 mV	B	inf	1.0	0.00153 V	0 V
BSt	20 mV	95.00	10.2 mV	B	inf	1.0	0.0102 V	0 V
TC	2 mV/deg C	95.00	1.02 mV/deg C	B	inf	5.60	0.00571 V	0 V/deg C
TRdegC	1.1 deg C	99.00	0.43 deg C	B	inf			5.6 deg C
DMMAcc	5.36 mV	95.00	2.73 mV	B	inf	1.0	0.00273 V	0 V
DMMres	0.5 mV	100.00	0.29 mV	B	inf	1.0	0.00029 V	0 V

Analysis Summary:
Computed Parameter Value: 4.859 V
Estimated Uncertainty: 87.8 mV
Distribution: Normal
Degrees of Freedom: infinite
Analysis Category: Type A,B

Parameter Value Equation:

```
'Overall Load Cell Calibration System Equation
SystemOut = LCOut * G + Gerror + DMMerror + Vran

'Load Cell Output Equation
LCOut = ((Wc + TEOut * TRdegF) * S + NL + Hys + NS + ZO + TEZero * TRdegF) * Vex

'Amplifier Error Equation
Gerror = GAcc + GS + GNL + GNS + BSt + TC * TRdegC

'Digital Multimeter Error Equation
DMMerror = DMMAcc + DMMres
```

Figure 7 UncertaintyAnalyzer Multivariate Analysis Summary Report

Uncertainty Analysis Report **Multivariate Uncertainty Analysis**

Pareto Diagram:

Rank	Error Component	Type	Weight (%)
1	Vex	B	49.315
2	ZO	B	26.267
3	BSt	B	6.567
4	TC	B	3.676
5	DMMAcc	B	1.758
6	Vran	A	1.706
7	GAcc	B	1.642
8	TEZero	B	1.313
9	NS	B	1.313
10	NL	B	1.313
11	Hys	B	1.313
12	Wc	B	1.195
13	GNS	B	0.985
14	TEOut	B	0.793
15	GNL	B	0.328
16	GS	B	0.328
17	DMMres	B	0.187

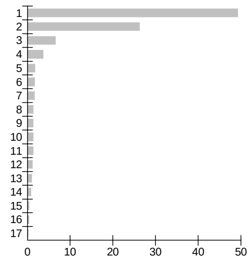


Figure 8 UncertaintyAnalyzer Multivariate Analysis Pareto Diagram

As expected, the analysis results obtained via Excel spreadsheet and UncertaintyAnalyzer are the same. In this case, it took over 4-1/2 hours to perform the necessary off-line calculations, develop the spreadsheet analysis and double-check the calculations. In contrast, it took less than 30 minutes to enter the necessary equations and error source data into UncertaintyAnalyzer.

<u>Simplified Root-Sum-Square Method</u>

Finally, let us examine the uncertainty analysis results obtained by simply combining the uncertainties of the various error sources in root-sum-square. This approach essentially assumes that all sensitivity coefficients have a value of unity. However, not all of the specifications (i.e., error limits) listed in Tables 1 through 3 and Tables 5 through 7 are in the desired mV units. Therefore, we must first convert the error limits as follows:

$$W_C \text{ error (mV)} = W_C \text{ error (lb)} \times S \text{ (mV/V/lb)} \times V_{ex} \text{ (V)} = \pm 0.0003 \times 0.4 \times 8 = \pm 0.00096 \text{ mV}$$

$$TE_{Out} \text{ (mV)} = TE_{Out} \text{ (lb/deg F)} \times S \times V_{ex} \times TR_{\circ F} = \pm 0.00015 \times 0.4 \times 8 \times 10 = \pm 0.0048 \text{ mV}$$

$$TE_{Zero} \text{ (mV)} = TE_{Zero} \text{ (mV/V/deg F)} \times TR_{\circ F} = \pm 0.0001 \times 10 = \pm 0.001 \text{ mV}$$

$$NL = NL \text{ (mV/V)} \times V_{ex} = \pm 0.001 \times 8 = \pm 0.008 \text{ mV}$$

$$HYS = HYS \text{ (mV/V)} \times V_{ex} = \pm 0.001 \times 8 = \pm 0.008 \text{ mV}$$

$$NS = NS \text{ (mV/V)} \times V_{ex} = \pm 0.001 \times 8 = \pm 0.008 \text{ mV}$$

$$ZO = ZO \text{ (mV/V)} \times V_{ex} = \pm 0.021 \times 8 = \pm 0.16 \text{ mV}$$

$$TC \text{ (mV)} = TC \text{ (mV/deg C)} \times TR_{\circ C} = \pm 2 \times 5.6 = \pm 11.2 \text{ mV}$$

The multivariate analysis spreadsheet was copied and then modified to include the above converted error limits and set all sensitivity coefficients equal to 1, as shown in Table 12.

By using a simplified root-sum-square combination method, the total uncertainty is significantly overestimated. The excitation voltage error appears to contribute almost all of the uncertainty. The effect of load cell zero offset error is not identified as significant contributor to total uncertainty. Consequently, not using the correct error propagation techniques, the estimated component uncertainties and total uncertainty do not reflect the actual measurement process. Unfortunately, this incorrect approach is often applied due to a genuine lack of understanding of error propagation principles and methods or as a means to minimize the time required to conduct an uncertainty analysis.

Fortunately, off-the-shelf applications such as UncertaintyAnalyzer provide proper analysis tools for computing realistic uncertainty estimates. In addition, these applications can significantly reduce the time and cost associated with the development and maintenance of in-house analysis spreadsheets.

Table 12 Uncertainty Analysis Results using Simplified RSS Method

				Parameter Value / Error Source Table									
Variable Name	Error Distribution	Error Limits	Units	% Conf.	Coverage Factor	Std Unc.	Units	Deg. Freedom	Sensitivity Coeff.	Component Unc.	Units	Nominal or Mean Value	Units
G				95								0.5	V/mV
Vran			mV			2.6458	mV	2	1	2.6458	mV	0.059	V
Wc	Normal	0.00096	mV	99	2.5758	0.0004	mV	infinite	1	0.0004	mV	3	lb
TEOut	Normal	0.0048	mV	95	1.9600	0.0024	mV	infinite	1	0.0024	mV	0	lb/deg F
TRdegF	Normal	2	deg F	99	2.5758	0.7764	deg. F	infinite	1			10	deg. F
S												0.4	mV/V/lb
NL	Normal	0.008	mV	95	1.9600	0.0041	mV	infinite	1	0.0041	mV	0	mV
Hys	Normal	0.008	mV	95	1.9600	0.0041	mV	infinite	1	0.0041	mV	0	mV
NS	Normal	0.008	mV	95	1.9600	0.0041	mV	infinite	1	0.0041	mV	0	mV
ZO	Normal	0.16	mV	95	1.9600	0.0816	mV	infinite	1	0.0816	mV	0	mV
TEZero	Normal	0.001	mV	95	1.9600	0.0005	mV	infinite	1	0.0005	mV	0	mV/deg F
Vex	Normal	0.25	V	95	1.9600	0.1276	V	infinite	1	127.5536	mV	8	V
GAcc	Normal	5	mV	95	1.9600	2.5511	mV	infinite	1	2.5511	mV	0	V
GS	Normal	1	mV	95	1.9600	0.5102	mV	infinite	1	0.5102	mV	0	V
GNL	Normal	1	mV	95	1.9600	0.5102	mV	infinite	1	0.5102	mV	0	V
GNS	Normal	3	mV	99	2.5758	1.1647	mV	infinite	1	1.1647	mV	0	V
BSt	Normal	20	mV	95	1.9600	10.2043	mV	infinite	1	10.2043	mV	0	V
TC	Normal	11.2	mV	95	1.9600	5.7144	mV	infinite	1	5.7144	mV	0	V/deg C
TRdegC	Normal	1.1	deg C	99	2.5758	0.4270	deg. C	infinite				5.6	deg. C
DMMAcc	Normal	5.36	mV	95	1.9600	2.7347	mV	infinite	1	2.7347	mV	0	V
DMMRes	Uniform	0.5	mV	100	1.7321	0.2887	mV	infinite	1	0.2887	mV	0	V
				System Output =		4.859	V	**Total Uncertainty =**		128.2	mV or	0.1282	V