RELIABILITY ANALYSIS METHODS FOR CALIBRATION INTERVALS: ANALYSIS OF TYPE III CENSORED DATA[1]

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ABSTRACT

Investigators attempting to find analysis techniques suitable for calibration interval analysis will discover that virtually nothing useful can be found in the reliability literature. This is primarily due to the fact that the various methodologies developed for "classical" reliability analysis are built around sampling plans in which unit failure times are known and recorded.

Unfortunately, calibration history data, on which calibration intervals are based, do not provide precise time to failure (i.e., out-of-tolerance) information. Consequently, methods are required which extend beyond classical reliability analysis techniques. This paper offers such an extension by providing a maximum likelihood estimation technique for the analysis of data characterized by unknown failure times. We label this data classification as Type III censored data, and note that it applies to calibration history data. The estimation method is illustrated using the exponential reliability function, Its extension to other distributions is fairly straightforward. Also reported is an approximate method which has yielded good results for the cases studied to date.

1.0   INTRODUCTION

Many facilities in the aerospace and defense industries attempt to arrive at test equipment calibration intervals applicable to homogeneous groups (e.g., model-manufacturer populations) of equipment. Most of us who have grappled with the problem agree that such intervals should be based on calibration history data taken over time. Approaches which follow this philosophy have as their cornerstone the surmise that the out-of-tolerance percentage of instrument populations increases with time elapsed since calibration. To illustrate this point, imagine that a group of like instruments, all of which are in-tolerance, are dispatched into ordinary service at some time $t = 0$. Imagine further that we could monitor the instruments at some time $t_1 > 0$ without disturbing the group in any way. We would expect, based on the physics of calibrateable attribute stabilities, that, for t_1 sufficiently large, some of the items would be found out-of-tolerance. Moreover, if the group could be re-examined at some time $t_2 > t_1$, we would expect still more items to be found out-of-tolerance, and so on.

From the foregoing, we might reasonable hope to discover the relationship between out-of-tolerance (or in-tolerance) percentage and time since calibration for a homogeneous group of instruments in a given usage and environmental setting by successive recalibrations, performed at various points in time. Specifically, by calibrating samples of items taken on the group at times $t_1, t_2, t_3, ... , t_m$, and obtaining out-of-tolerance percentages for these samples, we might be able to establish a functional or graphic relationship between group out-of-tolerance percentages and time elapsed since calibration. Having determined this relationship, we could then adjust the group's calibration interval to correspond to a measurement reliability target, or "in-tolerance" rate, commensurate with measurement assurance requirements. An interval established in this manner would be optimal in the sense that its determination is based on the most relevant information available and that its value is directly linked to reliability objectives.

The reliability literature abounds with techniques for determining such relationships for cases where equipment are continuously monitored to record unit failure times. These data are often used to estimate mean time between failure

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(MTBF) and other parameters from which predictions are made concerning equipment lifetimes, optimal maintenance intervals and the like. The various analysis tools developed for this purpose are typically grouped according to whether (1) monitoring stops after a certain time has elapsed or (2) after a given number of failures have been recorded. These monitoring alternatives are referred to as Type I and Type II censoring, respectively.

Since calibration history data are taken and recorded at periodic intervals of weeks, months or years, specific time at which instruments transition to an out-of-tolerance state (i.e. "fail") are unknown. Instead, such test data consist of intervals of time between calibrations together with information on how many failures occurred within these intervals. To distinguish this situation from that encountered elsewhere, we refer to monitoring schemes built around such unknown failure times as Type III censoring. With Type III censoring, whether monitoring stops after certain numbers of failures occur or certain elapsed times are reached is immaterial.

In this paper we present a basic methodology for analyzing Type III censored data. The method is particularly appropriate for the determination of calibration and test intervals for standards, test equipment and test systems. For purposes of discussion, we employ the exponential distribution to model the reliability function. In addition to its simplicity, results obtained using the exponential model are valid whether instruments are renewed (adjusted, repaired, etc.) at every calibration or are renewed only if found out-of-tolerance. For clarity of discussion, the present treatment is best viewed in the context of the former alternative. Extension to other distributions and to "renew if out-of-tolerance only" policies is fairly straightforward.

We initially discuss the nature of the data in some detail in Section 2. Section 3 deals with the estimation of the exponential failure rate parameter using the maximum likelihood technique and a simpler less precise approach. The application of the parameter estimate to the prediction of calibration and test intervals is covered in Section 4. And then finally, Section 5 presents several numerical examples.

2.0   TEST DATA

A population of instruments which undergo periodic testing will arrive at the testing lab at various times. It might be supposed that most of these times would be somewhat close to their assigned test interval, but such there is no guarantee that this is the case. Usually there will be several different values for the time elapsed since the last test. The value for the jth such time will be represented by t_j.

Several instruments could arrive at very close to the same elapsed time. The number of instruments arriving at or near the elapsed time t_i will be represented by n_j.

After testing each instrument, it is determined whether the instrument has failed or not. In calibration terms, this would correspond to whether the instrument was out-of-tolerance or not. The number of instruments being found in-tolerance with an elapsed time of t_i time will be represented by s_j.

A hypothetical set of data for a period of seven months is shown in Table 2.1. In this table, it can be seen that two ($n_6 = 2$) of the instruments were in the field for six months ($t_6 = 6$). Of these two instruments, only one was found to have passed its test ($s_6 = 1$).

TABLE 2.1
Test Data

j	t	n	s
1	1	1	1
2	2	3	3
3	3	1	1
4	4	2	2
5	5	5	4
6	6	2	1
7	7	1	1

3.0   ESTIMATION AND INFERENCE

In this section we develop the methodology for estimating the exponential failure rate parameter from test data. With the exponential distribution, the probability of an instrument not failing during an interval of length t_i is given by

$$\Pr(\text{No failure before } t_j) = e^{-\lambda t}, \tag{3.1}$$

where λ is the exponential failure rate parameter. In what follows, we develop an estimate for this parameter using both a simple approach and the maximum likelihood technique. Also, using certain properties of the maximum likelihood technique, we develop confidence limits for the estimates obtained.

3.1   A SIMPLE APPROACH

A simple approach to estimating the failure rate parameter λ, given in (3.1), is as follows. First, find the total number of instruments tested, N, and the number of passing instruments, S. These are given by

$$N = \sum_{j=1}^{m} n_j ,\qquad(3.1.1)$$

and

$$S = \sum_{j=1}^{m} s_j .\qquad(3.1.2)$$

Next, find the average amount of time between tests, T. This is a simple weighted average of the test times given by

$$T = \frac{1}{N} \sum_{j=1}^{m} n_j t_j .\qquad(3.1.3)$$

The proportion of failures during this average time T can be approximated by

$$\Pr(\text{No failures before } T) = S/N .\qquad(3/1/4)$$

Using equation (3.1), we can then write

$$S/N = e^{-\lambda_0 T} ,\qquad(3.1.5)$$

where λ_0 is our estimate of λ. Solving for (3.1.5) for λ_0 yields

$$\lambda_0 = -\frac{1}{T} \ln(S/N) .\qquad(3.1.6)$$

Applying this procedure to the data of Table 2.1, the total number of instruments tested is $N = 15$ with $S = 13$ of the instruments passing. The average time between tests, found using (3.1.3) is $T = 4.13333$. Inserting these values into equation (3.1.6) yields

$$\lambda_0 = -\frac{1}{4.13333} \ln(13./15) = 0.03462 .\qquad(3.1.7)$$

In the course of this investigation, estimates obtained in this way were surprisingly close to the more precise estimates obtained using the maximum likelihood technique which will be discussed in the next section.

3.2   MAXIMUM LIKELIHOOD ESTIMATE

In this section we develop the-maximum likelihood estimate of the exponential failure rate parameter λ, as given in (3.1). This method is not appropriate for hand calculation, but is simple to program on a desktop computer or programmable calculator.

We first develop a likelihood function which is basically an equation which gives the probability of having sampled our particular set of test data. The maximum likelihood technique requires us to maximize this likelihood with respect to our parameter of interest, which is the failure rate parameter λ. This will be done using a numerical technique (Newton-Raphson) since a closed form expression for the maximum likelihood estimate of λ is not available.

The probability of having s_j out of n_j instruments pass a test after an interval of length t_j is simply given by a binomial probability law. This is given mathematically by

$$\Pr(s_j \text{ passes}) = C_{s_j}^{n_j} \left(e^{-\lambda t_j}\right)^{s_j} \left(1 - e^{-\lambda t_j}\right)^{n_j - s_j} , \qquad (3.2.1)$$

where

$$C_{s_j}^{n_j} = \frac{n_j!}{(n_j - s_j)! s_j!} . \qquad (3.2.1)$$

Using this result, the likelihood for the test data described in Section 2.0 is given by

$$L = \prod_{j=1}^{m} C_{s_j}^{n_j} \left(e^{-\lambda t_j}\right)^{s_j} \left(1 - e^{-\lambda t_j}\right)^{n_j - s_j} . \qquad (3.2.2)$$

The maximum likelihood estimate is found by setting the derivative of the log of L with respect to λ equal to zero. This is given by

$$F(\lambda) = \sum_{j=1}^{m} \frac{(n_j - s_j) t_j}{\left(e^{\lambda t_j} - 1\right)} - s_j t_j = 0 . \qquad (3.2.3)$$

It is then necessary to solve (3.2.4) for λ to get the maximum likelihood estimate of λ. If we set $m = 1$, $n_1 = N$, $s_1 = S$ and $t_1 = T$, the solution for (3.2.4) is just λ_0 as given in (3.1.6). Unfortunately, equation (3.2.4) cannot be solved explicitly unless $m = 1$. Therefore, in most cases it will be necessary to solve for λ using an iterative technique such as the Newton-Raphson technique.

First we need an initial estimate of λ. The most likely candidate would be the simple approach estimate, λ_0 given in (3.1.6). It is then necessary to define the derivative of $F(\lambda)$ with respect to λ. This is given by

$$F'(\lambda) = -\sum_{j=1}^{m} \frac{(n_j - s_j) t_j^2}{\left(e^{\lambda t_j} - 1\right)^2} e^{\lambda t_j} . \qquad (3.2.4)$$

We then iteratively define the solution for λ by

$$\lambda_k = \lambda_{k-1} - F(\lambda_{k-1}) / F'(\lambda_{k-1}) . \qquad (3.2.5)$$

This process is continued until the relative change in λ_k is negligible, i.e., smaller than computer accuracy. For a FORTRAN single precision variable, this can be accomplished by iterating until

$$\frac{F(\lambda_k) / F'(\lambda_k)}{\lambda_k} \leq 0.000001 . \qquad (3.2.6)$$

The final value of λ_k is taken as the maximum likelihood estimate λ.

Using this procedure on the data of Table 2.1, the maximum likelihood estimate of λ is given by $\lambda = 0.03551$. This solution was found using the FORTRAN program given in the Appendix. It should be noted at this point that the estimate given by the simple approach, $\lambda_0 = 0.03462$, compares very favorably with the maximum likelihood estimate.

3.3   CONFIDENCE LIMITS

In this section, we will define confidence limits for the failure rate parameter estimates from the previous sections. In order to do this it will be necessary to derive equations for the estimated standard errors. In both the simple and maximum likelihood cases, the standard error estimate is obtained using maximum likelihood estimation theory methods. Then limits for the estimates will be given.

For the simple approach estimate, the standard error is calculated using the second derivative of the log-likelihood function F' given in (3.2.5) where $m = 1$, $n_1 = N$, $s_1 = S$ and $t_1 = T$ (N, S and T are defined in Section 3.1). This gives

$$F'(\lambda_0) = -\frac{(N-S)T^2}{\left(e^{\lambda_0 T} - 1\right)^2}e^{\lambda_0 T}.$$
(3.3.1)

The standard error for λ_0 is then given by

$$S_{\lambda_0} = \left[\frac{-1}{N\,F'(\lambda_0)}\right]^{1/2}.$$
(3.3.2)

The standard error could be used in the usual fashion to compute confidence limits by adding and subtracting a multiple of the standard error from the parameter estimate. However, this can result in obtaining negative lower limits. A simple means of getting around this problem is to use the exponent of the standard error which would be obtained if $\ln(\lambda)$ were estimated rather than λ. This results in limits which are strictly positive. If we define Z to be the $(1 - \alpha/2)\times100$th percentile of the normal distribution, then the upper and lower limits for a $(1-\alpha)\times100$ percent confidence interval for λ_0 are given by

$$\lambda_0^U = \lambda_0 e^{Z\,S_{\lambda_0}/\lambda_0},$$
(3.3.3)

and

$$\lambda_0^L = \lambda_0 e^{-Z\,S_{\lambda_0}/\lambda_0}.$$
(3.3.4)

For a 95 percent confidence level one would use $Z = 1.96$, while for a 90 percent confidence level one would use $Z = 1.645$.

For the data in Table 2.1, the F' function is

$$F'(\lambda_0) = -\frac{(15-13)(4.1333)^2}{\left[e^{(0.03462)(4.1333)} - 1\right]^2}e^{(0.03462)(4.1333)}$$
$$= -1665.8463.$$
(3.3.5)

Using (3.3.2), the standard error for λ_0 is then given by

$$S_{\lambda_0} = \left[\frac{-1}{(15)(01665.8463)}\right]^{1/2}$$
$$= 0.006326.$$
(3.3.6)

The upper and lower limits for a two-sided 90 percent level confidence interval are then computed using (3.3.3) and (3.3.4) to be

$$\lambda_0^U = (0.03462)e^{(1.645)(0.006326)/0.03462} = 0.04676,$$
(3.3.7)

and

$$\lambda_0^L = (0.03462)e^{-(1.645)(0.006326)/0.03462} = 0.02563.$$
(3.3.8)

The standard error and confidence limits for the maximum likelihood estimate λ are found in a similar manner. We use the function F' given in (3.2.5) to form the standard error of λ

$$S_\lambda = \left[\frac{-1}{N\,F'(\lambda)}\right]^{1/2}.$$
(3.3.9)

Defining Z as before, the upper and lower limits for $\alpha\,(1-\alpha)\times100$ percent confidence interval for λ are given by

$$\lambda^U = \lambda e^{Z\,S_\lambda/\lambda}$$
(3.3.10)

and

$$\lambda^L = \lambda e^{-Z\,S_\lambda/\lambda}$$
(3.3.11)

For the data in Table 2.1, the standard error of λ is 0.006494 as computed in the program described in the Appendix. Using equations (3.3.10) and (3.3.11), the upper and lower confidence limits for a 90 percent confidence interval are computed to be 0.04797 and 0.02629, respectively.

4.0   PREDICTING CALIBRATION INTERVALS

A major application of the analysis of periodic test data is in the prediction of optimum test intervals. In this section we develop an estimate of this interval using the results developed in Section 3.0.

The reliability of an instrument at the end of a test interval I is given by

$$R(I) = e^{-\lambda I} . \tag{4.1}$$

Solving (4.1) for I gives

$$I = -\frac{1}{\lambda} \ln(R) . \tag{4.2}$$

Therefore, a simple estimate of the test interval I is given by

$$i_0 = -\frac{1}{\lambda_0} \ln(R) , \tag{4.3}$$

where R is the reliability target. Using asymptotic methods, an approximate standard error for i_0 can be estimated using S_{λ_0} from (3.3.2) by

$$S_{i_0} = \frac{1}{\lambda_0^2} \left| \ln(R) \right| S_{\lambda_0} . \tag{4.4}$$

Upper and lower confidence limits for the predicted test interval can be defined using the confidence limits for λ_0 given in (3.3.3) and (3.3.4). The limits are

$$i_0^U = -\frac{1}{\lambda_0^L} \ln(R) \tag{4.5}$$

and

$$i_0^L = -\frac{1}{\lambda_0^U} \ln(R) . \tag{4.6}$$

Using the data of Table 2.1 with $\lambda_0 = 0.03462$, $S_{\lambda_0} = 0.006326$, $\lambda_0^U = 0.04676$ and $\lambda_0^L = 0.02563$ as given in Section 3, and $R = 0.85$, the simple estimate of the test interval along with its standard error and 90 percent confidence limits is given by

$$i_0 = -\ln(0.85)/0.03462 = 4.69 \tag{4.7}$$

$$S_{i_0} = \left| \ln(0.85) \right| (0.006326)/(0.03462)^2 = 0.8578 \tag{4.8}$$

$$i_0^U = -\ln(0.85)/0.02563 = 6.34 \tag{4.9}$$

$$i_0^L = -\ln(0.85)/0.04676 = 3.48 . \tag{4.10}$$

In a similar manner, the maximum likelihood estimate of the test interval is given by

$$i = -\ln(R)/\lambda , \tag{4.11}$$

and an approximate standard error of the estimate is given by

$$S_i = \left| \ln(R) \right| S_\lambda / \lambda^2 . \tag{4.12}$$

The associated upper and lower confidence limits for this estimate are defined using λ^U and λ^L as defined in (3.3.10) and (3.3.11). The limits are

$$i^U = -\ln(R)/\lambda^L , \tag{4.13}$$

and

$$i^L = -\ln(R)/\lambda^U .\qquad(4.14)$$

Using the data of Table 2.1 with $\lambda = 0.03551$, $S_\lambda = 0.006494$, $\lambda^U = 0.04797$ and $\lambda^L = 0.02629$ as given in Section 3, and $R = 0.85$, the maximum likelihood estimate of the test interval along with its standard error and 90 percent confidence limits is given by

$$i = -\ln(0.85)/0.03551 = 4.57\qquad(4.15)$$

$$S_i = \left|\ln(0.85)\right|(0.006494)/(0.03551)^2 = 0.8369\qquad(4.16)$$

$$i^U = -\ln(0.85)/0.02629 = 6.18\qquad(4.17)$$

$$i^L = -\ln(0.85)/0.04797 = 3.39 .\qquad(4.18)$$

5.0 EXAMPLES

In this section, we consider three numerical examples taken from hypothetical data. The calculations were performed by the FORTRAN program listed in the Appendix. All the results are developed using 90 percent confidence levels ($Z = 1.645$) and a target reliability of 0.85 ($R = 0.85$).

In the tables that follow, the data are listed followed by three columns of probability estimates. The first is the sample proportion p, given by

$$p_j = s_j/n_j .\qquad(4.19)$$

This is followed by the predicted reliability r_0 using the simple estimate and the predicted reliability r using the maximum likelihood estimate which are given by

$$r_j^0 = e^{-\lambda_0 t_j} ,\qquad(4.20)$$

and

$$r_j = e^{-\lambda t_j} .\qquad(4.20)$$

The data and probability estimates are followed by the simple estimates and confidence limits for the failure parameter λ and the test interval I. Then finally, the maximum likelihood estimates and confidence limits are printed. The first example shows the complete results for the data which were initially given in Table 2.1. These results are given in Table 5.1. It will be recalled that the simple estimates provided essentially the same analysis as the maximum likelihood analysis.

TABLE 5.1 COMPLETE ANALYSIS OF TABLE 2.1 DATA

Data

t	n	s	p	r_0	r
1	1	1	1.0000	0.9660	0.9651
2	3	3	1.0000	0.9331	0.9314
3	1	1	1.0000	0.9013	0.8989
4	2	2	1.0000	0.8707	0.8676
5	5	4	0.8000	0.8410	0.8373
6	2	1	0.5000	0.8124	0.8081
7	1	1	1.0000	0.7848	0.7799

Simple Estimates

Failure Rate	Standard Error	Upper Limit	Lower Limit
0.03462	0.006326	0.04676	0.02563

Test Interval	Standard Error	Upper Limit	Lower Limit
4.6942	0.8578	6.3403	3.4755

Maximum Likelihood Estimates

Failure Rate	Standard Error	Upper Limit	Lower Limit
0.03551	0.006494	0.04797	0.02629

Test Interval	Standard Error	Upper Limit	Lower Limit
4.5765	0.8369	6.1827	3.3876

The second example, shown in Table 5.2, represents a situation where very little data are available. It should be noted that with a small amount of data, such as in this example, the confidence limits will be wider than if there were more data available.

TABLE 5.2 EXAMPLE ANALYSIS WITH LITTLE DATA

Data

t	n	s	p	r_0	r
1	1	1	1.0000	0.9622	0.9606
2	1	1	1.0000	0.9258	0.9228
3	1	1	1.0000	0.8908	0.8864
4	1	1	1.0000	0.8571	0.8515
5	1	1	1.0000	0.8247	0.8179
6	1	0	0.0000	0.7936	0.7857
7	1	1	1.0000	0.7636	0.7548

Simple Estimates

Failure Rate	Standard Error	Upper Limit	Lower Limit
0.03854	0.014580	0.07181	0.02068

Test Interval	Standard Error	Upper Limit	Lower Limit
4.2171	1.5955	7.8579	2.2632

Maximum Likelihood Estimates

Failure Rate	Standard Error	Upper Limit	Lower Limit
0.04019	0.015228	0.07496	0.02155

Test Interval	Standard Error	Upper Limit	Lower Limit
4.0434	1.5320	7.5409	2.1680

The third example, shown in Table 5.3, portrays a situation where a great deal of data are available. In this situation the estimation of the parameter and the test interval is very accurate and the confidence intervals are very tight.

TABLE 5.3 EXAMPLE ANALYSIS WITH LOTS OF DATA

Data

t	n	s	p	r_0	r
1	1000	967	0.9670	0.9648	0.9640
2	1000	933	0.9330	0.9309	0.9293
3	1000	933	0.9330	0.8982	0.8959
4	1000	900	0.9000	0.8666	0.8636
5	1000	833	0.8330	0.8361	0.8325
6	1000	767	0.7670	0.8067	0.8026
7	1000	733	0.7330	0.7783	0.7737

Simple Estimates

Failure Rate	Standard Error	Upper Limit	Lower Limit
0.03580	0.000014	0.03583	0.03578

Test Interval	Standard Error	Upper Limit	Lower Limit
4.5393	0.0018	4.5422	4.5364

Maximum Likelihood Estimates

Failure Rate	Standard Error	Upper Limit	Lower Limit
0.03666	0.000014	0.03668	0.03664

Test Interval	Standard Error	Upper Limit	Lower Limit
4.4334	0.00174	4.4363	4.4306

In all three situations, the similarity between the simple estimate results and the full maximum likelihood results are remarkable. Although the conditions under which this similarity holds are not precisely known at present, it's quite possible that the simple estimate could be appropriate for many applications.

6.0 CONCLUSIONS

In this paper, we have presented a methodology which is applicable to the analysis of periodic test data. This method assumes that our knowledge is limited to the time elapsed since an instrument was last tested and the observation that the instrument either did or did not fail during this interval. The method is distinguished from classical methods which assume that failure times are known.

APPENDIX — TEST DATA ANALYSIS PROGRAM

```
      Program NCSL

C*********************************************
c
c     Type III Test Data Analysis Program
c
C*********************************************

c***  Declarations.

      Real t(100),n(100),S(100),i0,iU0,iL0,i,iU,iL
      Character File*26;
```

```
c***   Open the input file.

       write(6,*) 'NCSL TYPE III DATA ANALYSIS PROGRAM
       Write(6,*) 'Input file?'
       Read (5, '(a)') Fi I e
       Open(ll,File=File,Status='UNKNOWN')

c***    Read the data table.

       M = 0
c      Loop
10     Continue
           Read(11,'()',End=15) t(M+l),n(M+1),s(M+1)
           M = M + 1
           Go to 10
15     Continue
c      End Loop

c***   Calculate simple estimate.

       Sums = 0.0
       Sumt = 0.0
       Sumn = 0.0
       Do 20 j = 1,M
       Sums = Sums + s(j)
       Sumt = Sumt + t(j)*n(j)
       Sumn = Sumn + n(j)
20     Continue

       tbar = Sumt/Sumn
       b0 = -log(Sums/Sumn)/tbar

       DF - -1.0 * (Sumn - Sums)*tbar/(exp(b0*tbar) - 1.0)
       DF = DF*tbar*exp(b0*tbar)/(exp(b0*tbar) - 1.0)
       Sb0 = Sqrt(-l.0/Sumn/DF)

c***   Confidence interval for the simple estimate.

       bU0 = b0 * exp(1.645*Sb0/b0)
       bL0 - b0 * exp(-1.645*Sb0/b0)

c***   Calibration Interval Inference for the simple estimate.

       i0 = -log(0.85) / b0
       Si0 = Abs(log(0.85)/b0/b0)*Sb0
       iU0 = -1og(0.85) / bL0
       iL0 = -log(0.85) / bU0

c***   Maximum Likelihood Estimate.

       F = 1000.0
       DF = 1000.0
       b = b0 + F/DF

30     If (Abs(F/DF/b).gt.0.000001l) then
          b = b - F/DF
          F = 0.0
          DF = 0.0
          Do 40 j = 1,M
          Denom = (n(j) - s(j))*t(j)/(exp(b*t(j)) - 1.0)
           F = F + Denom - s(j)*t(j)
```

```
           DF = DF - Denom*t(j)*exp(b*t(j))/(exp(b*t(j)) - 1.0)
40         Continue
           Go to 30
        End If

        Sb = Sqrt(-l.0/Sumn/DF)

c***    Confidence interval for the maximum likelihood estimate.

        bU = b * exp(1.645*Sb/b)
        bL = b * exp(-1.645*Sb/b)

c***    Calibration Interval Inference.

        i = 1og(0.85) / b
        Si    Abs(log(0.85)/b/b)*Sb
        iU = -log(0.85) / bL
        iL = -log(0.85) / bU

c***    Print out the data.

        Write(6,'(''1''///t34,''Data''/)')
        Write(6,10000) 't','n','s','p','r0','r'
I0000 Format(' ',6(A10,1x)/)

        Do 50 j = 1,M
           p = s(j)/n(j)
           r0 = exp(-b0*t(j))
           r = exp(-b*t(j))
           Write (6, 2000) t(j),n(j),s(j),p,r0,r
20000     Format(' ',3(f10.0,lx),3(f10.4,lx))
50      Continue

c***    Print out the simple estimates.

        Write(6,*)
        Write(6,*) '                              Simple Estimates'

        Write(6,30000) 'b0   ','Standard Error','Upper Limit','Lower Limit'
30000 Format(/,' ',4(al5,lx))
        Write (6,40000) b0,Sb0,bU0,bL0
40000 Format(' ',4(fl5.8,lx))

        Write(6,30000) 'i0   ','Standard Error','Upper Limit','Lower Limit'
        Write (6,40000) i0,Si0,iU0,iL0

c***    Print out the maximum likelihood estimates.

Write(6,*)
Write(6,*)  '                              Maximum Likelihood Estimates'

Write(6,30000)'b    ','Standard Error','Upper Limit','Lower Limit'
Write(6,40000) b,Sb,bU,bL

Write(6,30000)'i    ','Standard Error','Upper Limit','Lower Limit'
Write(6,40000) i,Si,iU,iL

Write(6,'(''1'')')

Stop
End
```