Important Elements of an Uncertainty Analysis Report

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Abstract
Uncertainty analysis reports are an important aspect of measurement quality assurance. The analysis report should be readily understood and interpreted by others and comprehensive enough to provide practical guidance in the development of other uncertainty analyses.

This paper discusses key elements that should be included in an uncertainty analysis report, including those recommended in the GUM [1]. Recommended practices for using measurement and uncertainty units and decimal digits are also presented. An example analysis report is provided to illustrate how these elements can be combined to clearly indicate how an uncertainty analysis was conducted, what data and information were used, and any underlying assumptions that may have been applied.

1 Introduction

When reporting the results of an uncertainty analysis, Section 7 of the GUM recommends that the following information be included:

1. The estimated value of the quantity of interest (measurand) and the combined uncertainty and degrees of freedom of the estimate.
2. The functional relationship between the quantity of interest and the measured components, along with sensitivity coefficients, if applicable.
3. The value of each measurement component and its combined uncertainty and degrees of freedom.
4. A list of the measurement process uncertainties and associated degrees of freedom for each uncertainty, along with a description of how they were estimated.
5. A list of applicable correlation coefficients, including any cross-correlations between component uncertainties.

It is also a good practice to include a brief description of the measurement process, including the procedures and instrumentation used, and additional data, tables and plots that help clarify the analysis results. In reporting measurement results, it may be necessary to include confidence limits or an expanded uncertainty.

2 Report Elements

A comprehensive uncertainty analysis report should include the following elements:

- Measurement Process Overview

- Error Source List
- Error Equations
- Error Correlations
- Uncertainty Budget
- Combined Uncertainty and Degrees of Freedom
- Confidence Limits or Expanded Uncertainty (optional)
- Supporting Calculations
- Technical References

It is also important that the units and decimal digits used to report measured values, derived quantities and estimated uncertainties are consistent. Guidelines on the use of measurement and uncertainty units are provided in Section 3. Guidelines for the number of decimal digits used to report measured values, derived quantities and estimated uncertainties are presented in Section 4.

2.1 Measurement Process Overview

The physical quantity that is the subject of the uncertainty analysis should be clearly described. If this quantity is derived from the measurement of other quantities, then an equation that defines the mathematical relationship between the derived quantity of interest and the measured quantities should also be included.

The measurement process overview should also include a concise description of the measurement procedure, environmental conditions and the instruments, reference standards, or other equipment used. Repeat measurements, equipment specifications and other data used in the uncertainty analysis should be included in the main body of the report or in appendices. Alternatively, references to other documents or files containing this information should be appropriately cited in the report.

2.2 Error Source List

Measurement process errors are the basic elements of an uncertainty analysis. Therefore, it is important to list all errors that have been identified and evaluated in the uncertainty analysis. A brief description of each error should be provided, along with the appropriate error distribution, error containment limits and associated containment probability or confidence level. Alternatively, if an expanded uncertainty is specified for an error source, then it should be accompanied by a coverage factor.

2.3 Error Equations

The uncertainty analysis report should also include an error equation or model for each measured quantity. The error equation or model should provide an algebraic expression that defines the total error in the value of the quantity in terms of all relevant measurement process errors. In the case of multivariate measurements, an additional algebraic expression should be included that defines the error in the derived quantity in terms of the total errors (i.e., error components) in the measured quantities.

2.4 Error Correlations

Correlations between measurement process errors do not typically exist for directly measured quantities. However, instances may arise in multivariate measurements where cross-correlations exist between measurement process errors for different measured quantities. In this case, the pairs of correlated error sources and associated correlation coefficients should be reported.

2.5 Uncertainty Budget

An uncertainty budget provides a concise, tabulated summary of key information about the sources of measurement error and the associated uncertainties that contribute to the combined uncertainty in the measurement result. This tabulated analysis summary provides a useful tool for identifying significant contributors to measurement uncertainty and optimizing testing and calibration processes.

The following information should be included in an uncertainty budget table:

- Error source
- Error containment limits
- Error containment probability
- Error distribution
- Estimated uncertainty
- Estimate type (A or B)
- Degrees of freedom
- Sensitivity coefficient, if applicable (see below)
- Component uncertainty, if applicable (see below)

The first column contains all errors that have been identified and evaluated in the uncertainty analysis. The error containment limits, associated containment probability or confidence level and applicable error distribution are listed in the second through fourth columns. Columns five through seven contain the estimated uncertainty, estimate type and associated degrees of freedom.

For many direct measurements, the sensitivity coefficient for each error source has a value of unity and the component uncertainty is equivalent to the uncertainty. However, in multivariate measurements and some direct measurements, the sensitivity coefficients may not equal unity. In such cases, sensitivity coefficient and component uncertainty columns should be included in the uncertainty budget table.

In addition to the uncertainty budget table, it may be helpful to include plots, graphs or charts that provide further insight into the analysis results. A histogram plot shows the relative frequency of repeat measurements and provides an indication of data normality. A Pareto or bar chart displays the relative contribution of each uncertainty estimate to the overall, combined uncertainty.

2.6 Combined Uncertainty and Degrees of Freedom

The combined uncertainty for the measurement result and its degrees of freedom are an important element of any uncertainty analysis report. The degrees of freedom for the combined uncertainty are computed using the Welch-Satterthwaite formula[1] and reported as the nearest whole number.

> **Note**: For multivariate measurements, the value of each measurement component, its combined uncertainty and degrees of freedom should also be included.

It is good practice to include the equations used to compute the combined uncertainty and degrees of freedom. If correlations between pairs of errors exist, then a table listing the correlated errors and the associated correlation coefficients should also be included.

2.7 Confidence Limits and Expanded Uncertainty

In reporting measurement results, it may be useful to include an interval that contains the true value with some specified confidence level or probability. The interval may be reported as confidence limits, with an associated confidence level, or an expanded uncertainty, with an associated coverage factor [2].

2.8 Supporting Calculations

Sufficient detail should be provided for the calculation of all uncertainty estimates. Calculations may be performed manually or using validated uncertainty analysis software, spreadsheets or other technical applications.

If performed manually, then the inclusion of supporting equations and calculation steps should be included in an appendix of the uncertainty analysis report. These calculation steps

- should make it easier to verify the analysis results;
- can provide practical guidance in the development of other uncertainty analyses; and
- can be used in the validation of uncertainty analysis software, spreadsheets or other math and statistics applications.

2.9 Technical References

The report should also include references to equipment specification data sheets, operating manuals, technical papers, books, electronic data files and other sources of information used in the uncertainty analysis.

3 Measurement and Uncertainty Units

A measured value is generally expressed as a number and a unit. In this context, a unit is a particular physical quantity that has been defined and adopted by convention. For example, the International System of Units (SI) defines base units for seven physical quantities, listed in Table 1, that are assumed to be mutually independent.

[1] Methods for calculating the degrees of freedom for a combined uncertainty estimate for an error composed of correlated errors are available [3].

Table 1. SI Base Units[2]

Base Quantity	Unit Name	Unit Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

Units for other derived quantities, such as area, velocity and density, are defined in terms of the seven base quantities and units. Twenty-two derived quantities, listed in Table 2, have been given special unit names and symbols to simplify their expression.

Table 2. SI Coherent Derived Units[3]

Derived Quantity	Unit Name	Unit Symbol	Expressed in terms of SI Base Units	Expressed in terms of other SI Units
Plane Angle	radian	rad	m/m	
Solid Angle	steradian	sr	m^2/m^2	
Frequency	hertz	Hz	s^{-1}	
Force	newton	N	$m\ kg\ s^{-2}$	
Pressure, Stress	pascal	Pa	$m^{-1}\ kg\ s^{-2}$	N/m^2
Energy, Work, Amount of Heat	joule	J	$m^2\ kg\ s^{-2}$	N m
Power, Radiant Flux	watt	W	$m^2\ kg\ s^{-1}$	J/s
Electric Charge	coulomb	C	s A	
Electrical Potential Difference	volt	V	$m^2\ kg\ s^{-3}\ A^{-1}$	W/A
Capacitance	farad	F	$m^{-2}\ kg^{-1}\ s^4\ A^2$	C/V
Electric Resistance	ohm	Ω	$m^2\ kg\ s^{-3}\ A^{-2}$	V/A
Electric Conductance	siemens	S	$m^{-2}\ kg^{-1}\ s^3\ A^2$	A/V
Magnetic Flux	weber	Wb	$m^2\ kg\ s^{-2}\ A^{-1}$	V s
Magnetic Flux Density	tesla	T	$kg\ s^{-2}\ A^{-1}$	Wb/m^2
Inductance	henry	H	$m^2\ kg\ s^{-2}\ A^{-2}$	Wb/A
Celcius Temperature	degree Celcius	°C	K	
Luminous Flux	lumen	lm	cd sr	cd
Illuminance	lux	lx	m^{-2} cd	lm/m^2
Activity referred to a Radionuclide	becquerel	Bq	s^{-1}	

[2] *The International System of Units (SI)*, 8th edition, Bureau International des Poids et Mesures, 2006. See also, NIST Special Publication 811, 2008 Edition, *Guide for the Use of the International System of Units* (*SI*).

[3] These units are called *coherent derived* units because if only units from a coherent set are used, conversion factors between units are not required.

Derived Quantity	Unit Name	Unit Symbol	Expressed in terms of SI Base Units	Expressed in terms of other SI Units
Absorbed Dose	gray	Gy	$m^2 s^{-2}$	J/kg
Dose Equivalent	sievert	Sv	$m^2 s^{-2}$	J/kg
Catalytic Activity	katal	kat	s^{-1} mol	

The prefixes listed in Table 3 are used to represent unit multiples and unit subdivisions.

Table 3. SI Units Prefixes

Prefix	Symbol	Multiplying Factor	Prefix	Symbol	Multiplying Factor
deka	da	10^1	deci	d	10^{-1}
hecto	h	10^2	centi	c	10^{-2}
kilo	k	10^3	milli	m	10^{-3}
mega	M	10^6	micro	μ	10^{-6}
giga	G	10^9	nano	n	10^{-9}
tera	T	10^{12}	pico	p	10^{-12}
peta	P	10^{15}	femto	f	10^{-15}
exa	E	10^{18}	atto	a	10^{-18}
zetta	Z	10^{21}	zepto	z	10^{-21}
yotta	Y	10^{24}	yocto	y	10^{-24}

When reporting measurement uncertainty, it is a recommended practice to use either the same unit reported for the measured value or derived quantity or a subdivision of that unit. For example, if the value obtained from a pressure measurement is reported in kPa (kilopascal) units, then the associated measurement uncertainty should be reported in kPa or a subdivision unit such as Pa, mPa, μPa, etc.

It is not a good practice to report measured values and corresponding uncertainties using mixed units (e.g., W and dB). In some instances, it may be desirable to report uncertainty as a fraction, percent or ppm (parts per million) of the measured or derived value.[4] This "relative uncertainty" should be reported in addition to the measured value and associated uncertainty. For example, a measured mass value and uncertainty would be reported as 5.255 g with an uncertainty of 2.8 mg or 0.053%.

4 Decimal Digits

The number of decimal digits used in reporting measured values and uncertainty estimates should be informative, but not misleading. It is a recommended practice to base the decimal digits on the following factors:

- The resolution of the measuring equipment or device.
- The measurement units used.

[4] The accepted practice is to use ratios of units such as μg/g instead of ppm, nV/V instead of ppb, etc.

- The uncertainty units used.
- The information that needs to be conveyed.

For example, consider a dc voltage measurement made with a device with a range setting of 0 V to 10 V and a digital resolution of 1 mV. If a single measured value is obtained, then the measured value would be reported to three decimal digits (e.g., 5.001 V). An extra decimal digit may be justified when reporting an average value obtained from repeat measurements (e.g., 5.0014 V).

In either case, the number of decimal digits used to report measurement uncertainty should adequately convey the estimated value (e.g., 0.0006 V, 0.6 mV or 600 μV). In instances where the uncertainty is very small compared to the measured value, it may be beneficial to employ scientific notation (e.g., 6×10^{-4} V).

3 Example Analysis Report

An analysis report for the calibration of a relative humidity probe/indicator is provided in the following pages to illustrate how the report elements can be collated to clearly convey how an uncertainty analysis was conducted.

Relative Humidity Uncertainty Analysis Report
Acme Laboratory – March 10, 2009

Measurement Process Overview
The relative humidity (RH) measurement function of a Vaisala HMI41 Indicator/HMP46 High Temperature Probe is calibrated using a Thunder Scientific 9000 Automated Two-Pressure Humidity Generator [1]. The unit under test (UUT) probe/indicator is calibrated at three different relative humidity values.

The calibration result is the difference between the relative humidity measured by the UUT and the relative humidity measured by the humidity generator (MTE).

$$\delta = RH_{UUT} - RH_{MTE} \qquad (1)$$

The HMP46 probe has a specified accuracy of ± 1 %RH and the HMI41 indicator has a specified accuracy of ± 0.1 %RH [2]. The total accuracy specification limits are computed to be

$$\pm(1 + 0.1)\,\%\text{RH}$$
$$\pm 1.1\%\,\text{RH}$$

If the value of δ falls outside of the specified accuracy limits, then the UUT is typically deemed to be out-of-tolerance (OOT) or noncompliant. However, errors in the calibration process can result in an incorrect OOT assessment (false-reject) or incorrect in-tolerance assessment (false-accept). The relationship between the calibration result, δ, and the true UUT bias, $e_{UUT,b}$, is generally expressed as

$$\delta = e_{UUT,b} + \varepsilon_{cal}\,. \qquad (2)$$

The probability that the UUT is in-tolerance is based on the calibration result and its associated uncertainty. Therefore, all relevant calibration error sources must be identified and combined in a way that yields viable uncertainty estimates.

Uncertainty Analysis Procedure
The purpose of this analysis is to estimate and report the total uncertainty in δ for the relative humidity measurements listed in Table 1.

Table 1. Relative Humidity Calibration Data [3]

RH_{UUT} (% RH)	RH_{MTE} (% RH)	δ (% RH)	UUT Accuracy (% RH)
19.6	19.98	– 0.38	± 1.1
49.5	49.97	– 0.47	± 1.1
79.4	79.98	– 0.58	± 1.1

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The uncertainty in δ is determined by applying the variance operator to equation (2) and taking the square root.

$$u_\delta = \sqrt{\mathrm{var}(\delta)} = \sqrt{\mathrm{var}(e_{UUT,b} + \varepsilon_{cal})} = \sqrt{\mathrm{var}(\varepsilon_{cal})} \quad (3)$$

Note: In-depth coverage of key aspects of measurement uncertainty analysis methods used in this analysis is NASA-HNBK-8739.19.3 [4].

Given the equipment and procedures used, the calibration error equation is

$$\varepsilon_{cal} = \varepsilon_{MTE,bias} + \varepsilon_{MTE,res} + \varepsilon_{MTE,rep} + \varepsilon_{UUT,res} + \varepsilon_{UUT,rep} \quad (4)$$

where

$\varepsilon_{MTE,bias}$ = MTE bias
$\varepsilon_{MTE,res}$ = MTE resolution error
$\varepsilon_{MTE,rep}$ = MTE measurement repeatability
$\varepsilon_{UUT,res}$ = UUT resolution error
$\varepsilon_{UUT,rep}$ = UUT measurement repeatability

Note: Systematic contributions resulting from environmental factors or operator bias are not considered to be relevant to this analysis. Random contributions from these and other error sources are included in the measurement repeatability.

Brief descriptions of the calibration process errors are provided in the following paragraphs.

MTE Bias ($\varepsilon_{MTE,bias}$)

The relative humidity accuracy limits for the Thunder Scientific 9000 are specified to be 0.3 %RH [5]. The MTE accuracy limits are assumed to represent 95% confidence limits for a normally distributed error.

MTE Resolution Error ($\varepsilon_{MTE,res}$)

The digital display resolution of the Thunder Scientific 9000 is specified to be 0.01 %RH. The resolution error limits are ± 0.005 %RH (i.e., ± half the resolution). The resolution error limits represent 100% containment limits for a uniformly distributed error.

UUT Resolution Error ($\varepsilon_{UUT,res}$)

The Viasala MHI41 indicator has a digital display resolution of 0.1 %RH. The resolution error limits are ± 0.05 %RH. The resolution error limits represent 100% containment limits for a uniformly distributed error.

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Repeatability ($\varepsilon_{MTE,rep}$ and $\varepsilon_{UUT,rep}$)

The calibration procedure for the probe/indicator does not include steps for obtaining repeat measurements. A special test was conducted to assess the repeatability associated with the UUT and the humidity generator. Five repeat measurements were made at relative humidity calibration points of 20 %RH, 50 %RH and 80 %RH. The resulting data are listed in Table 2.

Table 2. Relative Humidity Repeatability Data [6]

Repeat Measurement	RH_{UUT} (% RH)	RH_{MTE} (% RH)	RH_{UUT} (% RH)	RH_{MTE} (% RH)	RH_{UUT} (% RH)	RH_{MTE} (% RH)
1	19.6	19.97	49.5	49.95	79.4	79.95
2	19.6	19.98	49.4	49.91	79.2	79.85
3	19.4	19.85	49.3	49.82	79.3	79.92
4	19.5	19.89	49.4	49.89	79.4	79.97
5	19.6	19.95	49.3	49.79	79.4	79.95
Average	19.54	19.928	49.38	49.872	79.34	79.928
Std Dev	0.09	0.056	0.08	0.066	0.09	0.047

Applying the variance operator to equation (4), and noting that there are no correlations between error sources, gives

$$\begin{aligned}\mathrm{var}(\varepsilon_{cal}) &= \mathrm{var}(\varepsilon_{MTE,bias}) + \mathrm{var}(\varepsilon_{MTE,res}) + \mathrm{var}(\varepsilon_{MTE,rep}) \\ &\quad + \mathrm{var}(\varepsilon_{UUT,res}) + \mathrm{var}(\varepsilon_{UUT,rep})\end{aligned} \tag{5}$$

The variance terms in equation (5) are equivalent to the square of the uncertainty in the corresponding error. So, the uncertainty equation for δ can be rewritten in terms of the individual measurement process uncertainties.

$$\begin{aligned}u_{\delta} &= \sqrt{\mathrm{var}(\varepsilon_{cal})} \\ &= \sqrt{u^2_{\varepsilon_{MTE,bias}} + u^2_{\varepsilon_{MTE,res}} + u^2_{\varepsilon_{MTE,rep}} + u^2_{\varepsilon_{UUT,res}} + u^2_{\varepsilon_{UUT,rep}}}\end{aligned} \tag{6}$$

The MTE bias uncertainty is estimated using the accuracy limits, the inverse normal distribution function and a 0.95 containment probability (i.e., 95% confidence level).

$$u_{\varepsilon_{MTE,bias}} = \frac{0.3\ \%\mathrm{RH}}{\Phi^{-1}\left(\dfrac{1+0.95}{2}\right)} = \frac{0.3\ \%\mathrm{RH}}{1.9600} = 0.15\ \%\mathrm{RH}$$

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The MTE resolution uncertainty is estimated using the resolution error limits, the inverse uniform distribution function and a 1.00 containment probability (100% confidence level).

$$u_{MTE,res} = \frac{0.005\ \%\text{RH}}{\sqrt{3}} = \frac{0.005\ \%\text{RH}}{1.732} = 0.0029\ \%\text{RH}$$

The UUT resolution uncertainty is estimated using the resolution error limits, the inverse uniform distribution function and a 1.00 containment probability (100% confidence level).

$$u_{UUT,res} = \frac{0.05\ \%\text{RH}}{\sqrt{3}} = \frac{0.05\ \%\text{RH}}{1.732} = 0.029\ \%\text{RH}$$

The MTE and UUT repeatability uncertainties are equal to the standard deviations of the repeat measurements. For example, at the 20% relative humidity calibration point,

$$u_{MTE,rep} = 0.056\ \%\text{RH} \text{ and } u_{UUT,rep} = 0.09\ \%\text{RH}.$$

An uncertainty budget summarizing the measurement process errors, distributions, uncertainties and degrees of freedom is given in Table 3.

Table 3. Uncertainty Budget for Humidity Probe/Indicator Calibration at 20 %RH

Error Source	Containment Limits (%RH)	Containment Probability (%)	Error Distribution	Standard Uncertainty (%RH)	Estimate Type	Deg. of Freedom
MTE Bias	± 0.3	95.00	Normal	0.15	B	∞
MTE Resolution	± 0.005	100.00	Uniform	0.0029	B	∞
MTE Repeatability				0.056	A	4
UUT Resolution	± 0.05	100.00	Uniform	0.029	B	∞
UUT Repeatability				0.09	A	4

The uncertainty in δ is computed by taking the root sum square of the process uncertainties. For a 20% relative humidity calibration point, the uncertainty in δ computed to be

$$u_\delta = \sqrt{(0.15\ \%\text{RH})^2 + (0.0029\ \%\text{RH})^2 + (0.056\ \%\text{RH})^2 + (0.029\ \%\text{RH})^2 + (0.09\ \%\text{RH})^2}$$
$$= \sqrt{0.035}\ \%\text{RH} = 0.19\ \%\text{RH}.$$

The Welch-Satterthwaite formula given in equation (7) is used to compute the degrees of freedom for u_δ.

$$\nu_{u_\delta} = \frac{u_\delta^4}{\dfrac{u_{\varepsilon_{MTE,bias}}^4}{\infty} + \dfrac{u_{\varepsilon_{MTE,res}}^4}{\infty} + \dfrac{u_{\varepsilon_{MTE,rep}}^4}{4} + \dfrac{u_{\varepsilon_{UUT,res}}^4}{\infty} + \dfrac{u_{\varepsilon_{UUT,rep}}^4}{4}} = 4 \times \frac{u_\delta^4}{u_{\varepsilon_{MTE,rep}}^4 + u_{\varepsilon_{UUT,rep}}^4} \quad (7)$$

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For a 20% relative humidity calibration point, the degrees of freedom for u_δ are computed to be

$$\nu_{u_\delta} = 4 \times \frac{(0.19\ \%\text{RH})^4}{(0.056\ \%\text{RH})^4 + (0.09\ \%\text{RH})^4} = 4 \times \frac{1.303 \times 10^{-3}}{9.835 \times 10^{-6} + 6.561 \times 10^{-5}}$$

$$= 4 \times \frac{1.303 \times 10^{-3}}{7.544 \times 10^{-5}} = 4 \times 17.3 = 69$$

The relative contributions of the measurement process uncertainties to the combined uncertainty are shown in Figure 1. The Pareto chart indicates that the MTE bias uncertainty is the largest contributor to the uncertainty in δ.

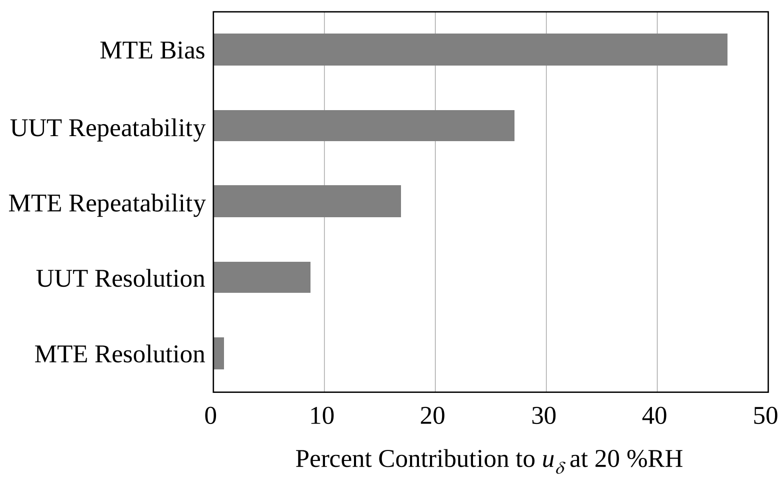

Figure 1 Pareto Chart for UUT Humidity Calibration Uncertainty at 20 %RH

The measurement results, combined uncertainty and associated degrees of freedom for the three calibration points of 20 %RH, 50 %RH and 80 %RH are summarized in Table 4. The largest value of δ is 0.58 %RH with an associated uncertainty of 0.18 %RH. This value of δ is an estimate of the UUT measurement bias, $e_{UUT,b}$, for a relative humidity of approximately 80 %RH at the time of calibration.

Table 4. Relative Humidity Uncertainty Analysis Results

RH_{UUT} (%RH)	RH_{MTE} (%RH)	δ (%RH)	Standard Uncertainty u_δ (%RH)	Degrees of Freedom	UUT Accuracy (%RH)
19.6	19.98	– 0.38	0.19	69	± 1.1
49.5	49.97	– 0.47	0.18	70	± 1.1
79.4	79.98	– 0.58	0.18	60	± 1.1

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References

1. Calibration Procedure for Vaisala HMI41 Indicator/HMP46 High Temperature Probe, ACME Laboratory, Document Number N50010, October 2005.
2. Vaisala HMI41 Indicator and HMP46 High Temperature Probe Technical Data Sheet, Vaisala Instruments Catalog Ref 8210712en rev A, Vaisala Corporation.
3. Calibration Data Sheet X123_Jan 15_2009, ACME Laboratory, Bakersfield, CA.
4. NASA-HNBK-8739.19-3 *Measurement Uncertainty Analysis Principles and Methods*, NASA Measurement Quality Assurance Handbook – Annex 3.
5. Thunder Scientific Model 9000 Automated “Two-Pressure” Humidity Generator, product data sheet 00298-9000, Thunder Scientific Corporation.
6. Repeatability data for Vaisala HMI41 Indicator/HMP46 High Temperature Probe, Humidity_Probe_2_12_09.xls, ACME Laboratory, 2/12/09.

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4 Conclusions

An uncertainty analysis report should be readily understood and interpreted by others. It should be comprehensive, and contain sufficient detail, so that the analysis results can be independently verified and reproduced. Such a comprehensive uncertainty analysis report can provide practical guidance in the development of other uncertainty analyses. Important elements of a comprehensive uncertainty analysis report have been discussed and illustrated.

5 References

1. ANSI/NCSL Z540-2-1997, the *U.S. Guide to the Expression of Uncertainty in Measurement*.

2. Castrup, S.: "Comparison of Methods for Establishing Confidence Limits and Expanded Uncertainties," Proc. of 2010 Measurement Science Conference, Pasadena, CA, March 2010.

3. Castrup, H.: "A Welch-Satterthwaite Relation for Correlated Errors," Proc. of 2010 Measurement Science Conference, Pasadena, CA, March 2010.

4. *The International System of Units (SI)*, 8th edition, Bureau International des Poids et Mesures, 2006.

5. NIST Special Publication 811, 2008 Edition, *Guide for the Use of the International System of Units (SI)*.