Applying Measuring and Test Equipment Specifications

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Abstract
Manufacturer specifications are an important element of cost and quality control for testing, calibration and other measurement processes. They are used in the selection of measuring and test equipment (MTE) and the establishment of equivalent equipment substitutions for a given measurement application.

MTE specifications are used to estimate measurement uncertainty, establish tolerance limits for calibration and testing, and evaluate false accept risk and false reject risk. MTE parameters are periodically calibrated to determine if they are performing within manufacturer specified tolerance limits. In fact, the elapsed-time or interval between calibrations is often based on in-tolerance or out-of-tolerance data acquired from periodic calibrations.

This paper provides illustrative examples of how MTE specifications are used to estimate parameter bias uncertainties, compute test tolerance limits, determine in-tolerance probability, and establish calibration intervals.

1   Introduction

In the fields of measurement science and metrology, MTE include artifacts, instruments, sensors and transducers, signal conditioners, data acquisition units, data processors and output displays. For the most part, manufacturer specifications are intended to convey tolerance or confidence limits that are expected to bound performance characteristics of MTE parameters or attributes. For example, these limits may correspond to temperature, shock and vibration parameters that affect the sensitivity and/or zero offset of a sensing device.

MTE users must become proficient at identifying relevant performance specifications and properly interpreting and applying them. Unfortunately, there is no universal guide or standard regarding the development and reporting of MTE specifications. Inconsistency in the methods used to develop and report performance specifications, and in the terms and units used to convey this information, create obstacles to the proper understanding and application of MTE specifications [1-4].

In select instances, the information included in a specification document may follow a standardized format.[1] However, the vast majority of specification documents fall short of providing crucial information about the confidence levels associated with reported specification limits. MTE manufacturers also don't indicate the applicable probability distribution for a

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[1] See for example, Ref [5].

particular performance characteristic.

1.1 Confidence Levels and Coverage Factors

Some MTE specifications are established by testing a selected sample of the produced model population. Since the test results are applied to the entire MTE model population, limits are developed to ensure that a large percentage of the MTE model population will perform as specified. Consequently, the specifications are confidence limits with associated confidence levels.[2]

Ideally, confidence levels should be commensurate with what MTE manufacturers consider to be the maximum allowable false accept risk (FAR).[3] The general requirement is to minimize the probability of shipping an MTE item with nonconforming (or out-of-compliance) performance characteristics. In this regard, the primary factor in setting the maximum allowable FAR may be the costs associated with shipping nonconforming products.

For example, an MTE manufacturer may require a maximum allowable FAR of 1% for all performance specifications. In this case, a 99% confidence level would be used to establish the MTE specification limits. Similarly, if the maximum allowable FAR is 5%, then a 95% confidence level should be used to establish the specification limits.

Alternatively, some manufacturers may test the entire produced MTE model population to ensure that individual items are performing within specified limits prior to shipment. However, this compliance testing process does not ensure a 100% probability (or confidence level) that the customer will receive an in-tolerance item. The reasons for this include

1. Measurement uncertainty associated with the manufacturer MTE compliance testing process.

2. MTE bias drift or shift resulting from shock, vibration and other environmental extremes during shipping and handling.

Manufacturers may attempt to mitigate this problem by increasing the MTE specification limits. This can be accomplished by using a higher confidence level (e.g., 99.9%) to establish larger specification limits. Alternatively, some manufacturers may employ arbitrary guardbanding[4] methods and multiplying factors. In either case, the resulting MTE specifications are not equivalent to 100% confidence limits.

The criteria and motives used by manufacturers to establish MTE specifications are not often apparent. Most MTE manufacturers see the benefits, to themselves and their customers, of establishing specifications with high confidence levels. However, "specsmanship" between MTE manufacturers can result in tighter specifications and increased out-of-tolerance occurrences.[5]

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[2] In this context, confidence level and containment probability are synonymous, as are confidence limits and containment limits.

[3] See Ref [6] for a discussion on false accept risk.

[4] Guardbands are supplemental limits used to reduce false accept risk.

[5] See for example, Ref [7].

1.2 Probability Distributions

MTE performance characteristics, such as nonlinearity, repeatability, hysteresis, resolution, noise, thermal stability and zero shift constitute sources of measurement error. Measurement errors are random variables that can be characterized by probability distributions. Therefore, MTE performance characteristics are also considered to be random variables that follow probability distributions.

The probability distribution for a type of measurement error is a mathematical description that relates the frequency of occurrence of values to the values themselves. Error distributions include, but are not limited to normal, lognormal, uniform (rectangular), triangular, quadratic, cosine, exponential and u-shaped.

This concept is important to the interpretation and application of MTE specifications because an error distribution allows us to determine the probability that a performance characteristic is in conformance or compliance with its specification.

2 Measurement Uncertainty Analysis

Manufacturer specifications can be used to conduct a preliminary assessment of the uncertainty in the nominal value or output of the MTE. The analysis results can then be used to identify, and possibly mitigate, the largest contributors to overall uncertainty. These preliminary analyses can be (and should be) conducted before MTE are selected or purchased.

For illustration, an uncertainty analysis is conducted for a strain-gage based load cell with the basic transfer function[6] given in equation (1) and manufacturer specifications listed in Table 1.

$$LC_{out} = W \times S \times V_{Ex} \tag{1}$$

where

LC_{out} = Output voltage
W = Applied load or weight
S = Load cell sensitivity
V_{Ex} = Excitation voltage

Table 1. Load Cell Specifications

Specification	Value	Units
Rated Output (R.O.)	2 (nominal)	mV/V
Maximum Load	5	lb$_f$
Nonlinearity	0.05% of R.O.	mV/V
Hysteresis	0.05% of R.O.	mV/V
Nonrepeatability (Noise)	0.05% of R.O.	mV/V
Zero Balance (Zero Offset)	1.0% of R.O.	mV/V
Compensated Temp. Range	60 to 160	°F
Temperature Effect on Output	0.005% of Load/°F	lb$_f$/°F
Temperature Effect on Zero	0.005% of R.O./°F	mV/V/°F
Required Excitation	10	V$_{DC}$

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[6] A transfer function describes the mathematical relationship between the input and output response of the measuring device.

The load cell has a rated output of 2 mV/V for loads up to 5 lb_f which equates to a nominal sensitivity of 0.4 mV/V/lb_f. According to the specifications, the load cell output will be affected by the following performance parameters[7]:

- Excitation Voltage
- Nonlinearity
- Hysteresis
- Noise
- Zero Offset
- Temperature Effect on Output
- Temperature Effect on Zero

Equation (1) must be modified to account for these parameters. Given the assortment of specification units, it is apparent that the parameters cannot simply be added at the end of the equation. The appropriate load cell output equation is expressed in equation (2).

$$LC_{out} = \left[\left(W + TE_{out} \times TR_{°F} \right) \times S + NL + Hys + NS + ZO + TE_{zero} \times TR_{°F} \right] \times V_{Ex} \qquad (2)$$

where

LC_{out} = Output voltage, mV
W = Applied weight or load, lb_f
TE_{out} = Temperature effect on output, lb_f/°F
$TR_{°F}$ = Temperature range, °F
S = Load cell sensitivity, mV/V/lb_f
NL = Nonlinearity, mV/V
Hys = Hysteresis, mV/V
NS = Noise and ripple, mV/V
ZO = Zero offset, mV/V
TE_{zero} = Temperature effect on zero, mV/V/°F
V_{Ex} = Excitation Voltage, V

Given some knowledge about the load cell parameters and their associated probability distributions, the uncertainty in the load cell output voltage can be estimated. A 3 lb_f applied load is used for this analysis.

Excitation Voltage (V_{Ex})
Since the load cell is a passive sensor, it requires an external power supply. An 8 V_{DC} external power source is supplied to the load cell. The excitation voltage has ± 0.25 V error limits, which are interpreted to be 95% confidence limits for a normally distributed error.

Nonlinearity (*NL*)
Nonlinearity is a measure of the deviation of the actual input-to-output performance of the load cell from an ideal linear relationship. Nonlinearity error is fixed at any given input, but varies with magnitude and sign over a range of inputs. Therefore, it is considered to be a random error

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[7] If the load cell is tested or calibrated using a weight standard, then any error associated with the weight must also be evaluated.

that is normally distributed. The manufacturer specification of $\pm\,0.05\%$ of the rated output is interpreted to be 95% confidence limits.

Hysteresis (*Hys*)
Hysteresis indicates that the output of the load cell is dependent upon the direction and magnitude by which the input is changed. At any input value, hysteresis can be expressed as the difference between the ascending and descending outputs. Hysteresis error is fixed at any given input, but varies with magnitude and sign over a range of inputs. Therefore, it is considered to be a random error that is normally distributed. The manufacturer specification of $\pm\,0.05\%$ of the rated output is interpreted to be 95% confidence limits.

Noise (*NS*)
Noise is the nonrepeatability or random error intrinsic to the load cell that causes the output to vary from observation to observation for a constant input. This error source varies with magnitude and sign over a range of inputs and is normally distributed. The manufacturer specification of $\pm\,0.05\%$ of the rated output is interpreted to be 95% confidence limits.

Zero Offset (*ZO*)
Zero offset occurs if the load cell generates a non-zero output for a zero applied load. Making an adjustment to reduce zero offset does not eliminate the associated error because there is no way to know the true value of the offset. The manufacturer specification of $\pm\,1\%$ of the rated output is interpreted to be 95% confidence limits for a normally distributed error.

Temperature Effects (*TE$_{out}$* and *TE$_{zero}$*)
Temperature can affect both the offset and sensitivity of the load cell. To establish these effects, the load cell is typically tested at several temperatures within its operating range and the effects on zero and sensitivity or output are observed.

The temperature effect on output of 0.005% load/°F specified by the manufacturer is equivalent to 0.00015 lb/°F for an applied load of 3 lb$_f$. The temperature effect on zero specification of 0.005% R.O./°F and the temperature effect on output are interpreted to be 95% confidence limits for normally distributed errors.

The load cell is part of a tension testing machine, which heats up during use. The load cell temperature is monitored and recorded during the testing process and observed to increase from 75 °F to 85 °F. For this analysis, the 10 °F temperature range is assumed to have error limits of $\pm\,2$ °F with an associated 99% confidence level. The temperature measurement error is also assumed to be normally distributed.

The parameters used in the load cell output equation are listed in Table 2. The normal distribution is applied for all parameters. The error model for the load cell output is given in equation (3).

$$\varepsilon_{LC_{out}} = c_S \varepsilon_S + c_{NL} \varepsilon_{NL} + c_{Hys} \varepsilon_{Hys} + c_{NS} \varepsilon_{NS} + c_{ZO} \varepsilon_{ZO} + c_{TE_{out}} \varepsilon_{TE_{out}}$$
$$+ c_{TE_{zero}} \varepsilon_{TE_{zero}} + c_{TR_{°F}} \varepsilon_{TR_{°F}} + c_{V_{Ex}} \varepsilon_{V_{Ex}}$$

(3)

Table 2. Parameters used in Load Cell Output Equation

Parameter Name	Description	Nominal or Stated Value	Error Limits	Confidence Level
W	Applied Load	3 lb$_f$		
S	Load Cell Sensitivity	0.4 mV/V/lb$_f$		
NL	Nonlinearity	0 mV/V	$\pm$ 0.05% R.O. (mV/V)	95
Hys	Hysteresis	0 mV/V	$\pm$ 0.05% R.O. (mV/V)	95
NS	Nonrepeatability	0 mV/V	$\pm$ 0.05% R.O. (mV/V)	95
ZO	Zero Offset	0 mV/V	$\pm$ 1% R.O. (mV/V)	95
$TR_{\circ F}$	Temperature Range	10 °F	$\pm$ 2.0 °F	99
TE_{out}	Temp Effect on Output	0 lb$_f$/°F	$\pm$ 0.005% Load/°F (lb$_f$/°F)	95
TE_{zero}	Temp Effect on Zero	0 mV/V /°F	$\pm$ 0.005% R.O./°F (mV/V/°F)	95
V_{Ex}	Excitation Voltage	8 V	$\pm$ 0.250 V	95

The coefficients in equation (3) are sensitivity coefficients that determine the relative contribution of the individual errors to the total error in LC_{out}. The partial derivative equations used to compute the sensitivity coefficients are listed below.

$$c_S = \frac{\partial LC_{out}}{\partial S} = \left(W + TE_{out} \times TR_{\circ F}\right) \times V_{Ex} \qquad c_{NL} = \frac{\partial LC_{out}}{\partial NL} = V_{Ex} \qquad c_{Hys} = \frac{\partial LC_{out}}{\partial Hys} = V_{Ex}$$

$$c_{NS} = \frac{\partial LC_{out}}{\partial NS} = V_{Ex} \qquad c_{ZO} = \frac{\partial LC_{out}}{\partial ZO} = V_{Ex} \qquad c_{TE_{out}} = \frac{\partial LC_{out}}{\partial TE_{out}} = TR_{\circ F} \times S \times V_{Ex}$$

$$c_{TE_{zero}} = \frac{\partial LC_{out}}{\partial TE_{zero}} = TR_{\circ F} \times V_{Ex} \qquad c_{TR_{\circ F}} = \frac{\partial LC_{out}}{\partial TR_{\circ F}} = \left(TE_{out} \times S + TE_{zero} \right) \times V_{Ex}$$

$$c_{V_{Ex}} = \frac{\partial LC_{out}}{\partial V_{Ex}} = \left(W + TE_{out} \times TR_{\circ F}\right) \times S + NL + Hys + NS + ZO + TE_{zero} \times TR_{\circ F}$$

Measurement uncertainty is the square root of the variance of the error distribution.[8] This means that the uncertainty in the load cell output can be computed from

$$u_{LC_{out}} = \sqrt{\mathrm{var}\left(\varepsilon_{LC_{out}}\right)} \qquad\qquad (4)$$

where var() is the variance operator.

Applying the variance operator to equation (3), and noting that there are no correlations between errors, gives

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[8] see Ref [8] for a basic discussion of the mathematical relationship between error and uncertainty.

$$\text{var}\left(\varepsilon_{LC_{out}}\right) = c_S^2 \, \text{var}\left(\varepsilon_S\right) + c_{NL}^2 \, \text{var}\left(\varepsilon_{NL}\right) + c_{Hys}^2 \, \text{var}\left(\varepsilon_{Hys}\right) + c_{NS}^2 \, \text{var}\left(\varepsilon_{NS}\right)$$

$$+ c_{ZO}^2 \, \text{var}\left(\varepsilon_{ZO}\right) + c_{TE_{out}}^2 \, \text{var}\left(\varepsilon_{TE_{out}}\right) + c_{TE_{zero}}^2 \, \text{var}\left(\varepsilon_{TE_{zero}}\right) \qquad (5)$$

$$+ c_{TR\circ_F}^2 \, \text{var}\left(\varepsilon_{TR\circ_F}\right) + c_{V_{Ex}}^2 \, \text{var}\left(\varepsilon_{V_{Ex}}\right)$$

The variance terms in equation (5) are equivalent to the square of the uncertainty in the corresponding error. Therefore, the uncertainty in the load cell output can be expressed as

$$u_{LC_{out}} = \sqrt{\begin{array}{l} c_S^2 u_S^2 + c_{NL}^2 u_{NL}^2 + c_{Hys}^2 u_{Hys}^2 + c_{NS}^2 u_{NS}^2 + c_{ZO}^2 u_{ZO}^2 + c_{TE_{out}}^2 u_{TE_{out}}^2 \\ + c_{TE_{zero}}^2 u_{TE_{zero}}^2 + c_{TR\circ_F}^2 u_{TR\circ_F}^2 + c_{V_{Ex}}^2 u_{V_{Ex}}^2 \end{array}} \qquad (6)$$

The uncertainty in the load cell output is computed from the uncertainty estimates and sensitivity coefficients for each load cell parameter.

As previously discussed, all of the errors identified in the load cell output equation are assumed to follow a normal distribution. Therefore, the corresponding uncertainties are estimated from the error limits, $\pm L$, confidence level, p, and the inverse normal distribution function, $\Phi^{-1}()$.

$$u = \frac{L}{\Phi^{-1}\left(\dfrac{1+p}{2}\right)} \qquad (7)$$

For example, the uncertainty in the excitation voltage error is estimated to be

$$u_{V_{Ex}} = \frac{0.25 \text{ V}}{\Phi^{-1}\left(\dfrac{1+0.95}{2}\right)} = \frac{0.25 \text{ V}}{1.9600} = 0.1276 \text{ V}.$$

The sensitivity coefficients are computed using the parameter nominal or stated values.

$$c_S = \left(W + TE_{out} \times TR\circ_F\right) \times V_{Ex} = \left(3 \text{ lb}_f + 0 \text{ lb}_f \,/\,°F \times 10 \,°F\right) \times 8 \text{ V}$$
$$= 3 \text{ lb}_f \times 8 \text{ V} = 24 \text{ lb}_f \bullet \text{V}$$

$$c_{NL} = V_{Ex} = 8 \text{ V} \qquad c_{Hys} = V_{Ex} = 8 \text{ V} \qquad c_{NS} = V_{Ex} = 8 \text{ V} \qquad c_{ZO} = V_{Ex} = 8 \text{ V}$$

$$c_{TR\circ_F} = \left(TE_{out} \times S + TE_{zero}\right) \times V_{Ex} = \left(0 \times 0.4 \text{ mV/V/lb}_f + 0\right) \times 8 \text{ V} = 0$$

$$c_{TE_{out}} = TR\circ_F \times S \times V_{Ex} = 10 \,°F \times 0.4 \text{ mV/V/lb}_f \times 8 \text{ V} = 32 \,°F \bullet \text{mV/lb}_f$$

$$c_{TE_{zero}} = TR_{\circ F} \times V_{Ex} = 10 \text{ °F} \times 8 \text{ V} = 80 \text{ °F} \bullet \text{V}$$

$$
\begin{aligned}
c_{V_{Ex}} &= \left(W + TE_{out} \times TR_{\circ F}\right) \times S + NL + Hys + NS + ZO + TE_{zero} \times TR_{\circ F} \\
&= \left(3 \text{ lb}_f + 0 \text{ lb}_f/\text{°F} \times 10 \text{ °F}\right) \times 0.4 \text{ mV/V/lb}_f + 0 \text{ mV/V} + 0 \text{ mV/V} + 0 \text{ mV/V} + 0 \text{ mV/V} \\
&\quad + 0 \text{ mV/V/°F} \times 10 \text{ °F} \\
&= 3 \text{ lb}_f \times 0.4 \text{ mV/V/lb}_f = 1.2 \text{ mV/V}
\end{aligned}
$$

The estimated uncertainties and sensitivity coefficients for each parameter are listed in Table 3.

Table 3. Estimated Uncertainties for Load Cell Parameters

Param. Name	Nominal or Stated Value	± Error Limits	Conf. Level	Standard Uncertainty	Sensitivity Coefficient	Component Uncertainty
W	3 lb$_f$					
S	0.4 mV/V/lb$_f$				24 lb$_f$ • V	
NL	0 mV/V	± 0.001 mV/V	95	0.0005 mV/V	8 V	0.0041 mV
Hys	0 mV/V	± 0.001 mV/V	95	0.0005 mV/V	8 V	0.0041 mV
NS	0 mV/V	± 0.001 mV/V	95	0.0005 mV/V	8 V	0.0041 mV
ZO	0 mV/V	± 0.02 mV/V	95	0.0102 mV/V	8 V	0.0816 mV
$TR_{\circ F}$	10 °F	± 2.0 °F	99	0.7764 °F	0	
TE_{out}	0 lb$_f$/°F	± 1.5 × 10^{-4} lb$_f$/°F	95	7.65 × 10^{-5} lb$_f$/°F	32 °F • mV/lb$_f$	0.0024 mV
TE_{zero}	0 mV/°F	± 0.0001 mV/V/°F	95	5 × 10^{-5} mV/V/°F	80 °F • V	0.0041 mV
V_{Ex}	8 V	± 0.25 V	95	0.1276 V	1.2 mV/V	0.1531 mV

The component uncertainties listed in Table 3 are the products of the standard uncertainty and sensitivity coefficient for each parameter. The nominal load cell output is computed to be

$$LC_{out} = W \times S \times V_{Ex} = 3 \text{ lb}_f \times 0.4 \text{ mV/V/lb}_f \times 8 \text{ V} = 9.6 \text{ mV}.$$

The total uncertainty in the load cell output is computed by taking the root sum square of the component uncertainties.

$$
\begin{aligned}
u_{LC_{out}} &= \sqrt{\begin{array}{l}(0.0041 \text{ mV})^2 + (0.0041 \text{ mV})^2 + (0.0041 \text{ mV})^2 + (0.0816 \text{ mV})^2 \\ + (0.0024 \text{ mV})^2 + (0.0041 \text{ mV})^2 + (0.1531 \text{ mV})^2\end{array}} \\
&= \sqrt{0.0302 \text{ mV}^2} = 0.174 \text{ mV}
\end{aligned}
$$

The total uncertainty is equal to 1.8% of the 9.60 mV load cell output. The Welch-Satterthwaite formula[9] is used to compute the degrees of freedom for the uncertainty in the load cell output voltage, as shown in equation (8).

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[9] A discussion and derivation of the Welch-Satterthwaite formula is given in Ref [9].

$$v_{u_{LC_{out}}} = \cfrac{u_{LC_{out}}^4}{\left[\cfrac{c_{NL}^4 u_{NL}^4}{v_{NL}} + \cfrac{c_{Hys}^4 u_{Hys}^4}{v_{Hys}} + \cfrac{c_{NS}^4 u_{NS}^4}{v_{NS}} + \cfrac{c_{ZO}^4 u_{ZO}^4}{v_{ZO}} + \cfrac{c_{TR\circ F}^4 u_{TR\circ F}^4}{v_{TR\circ F}} + \cfrac{c_{TE_{out}}^4 u_{TE_{out}}^4}{v_{TE_{out}}} \atop + \cfrac{c_{TE_{zero}}^4 u_{TE_{zero}}^4}{v_{TE_{zero}}} + \cfrac{c_{V_{Ex}}^4 u_{V_{Ex}}^4}{v_{V_{Ex}}} \right]} \tag{8}$$

The degrees of freedom for all of the error source uncertainties are assumed infinite. Therefore, the degrees of freedom for the uncertainty in the load cell output is also infinite.

The pareto chart, shown in Figure 1, indicates that the excitation voltage and zero offset errors are, by far, the largest contributors to the overall uncertainty the load cell output. Replacement of the power supply with a precision voltage source could significantly reduce the total uncertainty in the load cell output. Mitigation of the zero offset error, however, would probably require a different model load cell.
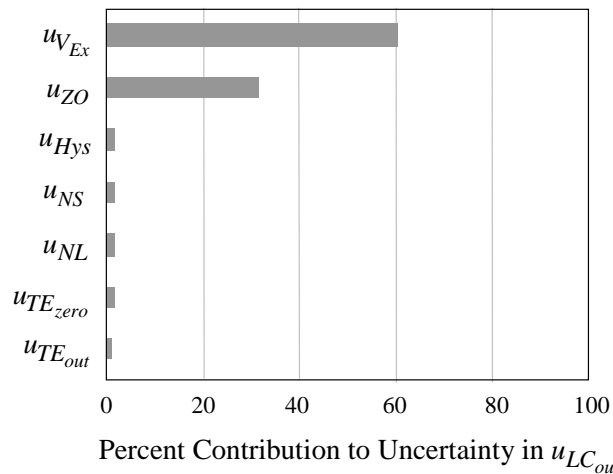

Figure 1 Pareto Chart for Uncertainty in Load Cell Output

The results of this analysis show that manufacturer specifications can be used to estimate the expected uncertainty in the load cell output and to identify the major contributors to this uncertainty.

3   Tolerance Limits

The load cell output uncertainty and degrees of freedom, v, can be used to compute confidence limits that are expected to contain the output voltage with some specified confidence level or probability, p. The confidence limits are expressed as

$$LC_{out} \pm t_{\alpha/2,v} \times u_{LC_{out}} \tag{9}$$

where the multiplier, $t_{\alpha/2,v}$, is the t-statistic and $\alpha = 1 - p$.

Tolerance limits for a 95% confidence level (i.e., $p = 0.95$) are computed using a corresponding t-statistic of $t_{0.025,\infty} = 1.96$.

$$9.60 \text{ mV} \pm 1.96 \times 0.174 \text{ mV} \text{ or } 9.60 \text{ mV} \pm 0.341 \text{ mV}$$

The uncertainty analysis and tolerancing processes can be repeated for different applied loads. The results are summarized in Table 4.

Table 4. Uncertainty in Load Cell Output Voltage versus Applied Load

Applied Load	Output Voltage	Total Uncertainty	Confidence Limits (95%)
1 lb$_f$	3.2 mV	0.097 mV	$\pm$ 0.190 mV
2 lb$_f$	6.4 mV	0.131 mV	$\pm$ 0.257 mV
3 lb$_f$	9.6 mV	0.174 mV	$\pm$ 0.341 mV
4 lb$_f$	12.8 mV	0.220 mV	$\pm$ 0.431 mV
5 lb$_f$	16.0 mV	0.268 mV	$\pm$ 0.525 mV

The *Slope* and *Intercept* functions of the Microsoft Excel application can then be used to obtain a linear fit of the above tolerance limits as a function of the applied load. These calculations yield an intercept value of 0.095 mV and a slope value of 0.084 mV/lb$_f$. From these values, the 95% confidence limits can be written as

$$\pm (0.095 \text{ mV} + 0.084 \text{ mV/lb}_f \times \text{Applied Load}).$$

As shown in Figure 1, the uncertainty in the excitation voltage has a significant impact on the confidence limits computed in this load cell example. These limits are also influenced, to a lesser extent, by the temperature range that the load cell is exposed to during use.

4   Measurement Decision Risk Analysis

The probability of making an incorrect decision based on a measurement result is called measurement decision risk. ANSI/NCSLI Z540.3:2006, Section 5.2 states that "b) Where calibrations provide for verification that measurement quantities are within specified tolerances, the probability that incorrect acceptance decisions (false accept) will result from calibration tests shall not exceed 2% and shall be documented." [10]

Various probability concepts and definitions are employed in the computation of measurement decision risk [6]. For instance, the probability that an MTE parameter or attribute accepted during calibration and testing as being in-tolerance is actually out-of-tolerance (OOT) is called false accept risk (FAR). Conversely, the probability that an MTE parameter determined to be OOT is actually in-tolerance is called false reject risk (FRR).

The primary purpose of calibration is to obtain an estimate of the value or bias of MTE attributes or parameters. Another important purpose it to ascertain the conformance or non-conformance of MTE parameters with specified tolerance limits. The calibration result, δ, is taken to be an estimation of the true parameter bias, $e_{UUT,b}$, of the unit under test (UUT) [11]. The relationship between δ and $e_{UUT,b}$ is generally expressed as

$$\delta = e_{UUT,b} + \varepsilon_{cal} \tag{10}$$

where ε_{cal} is the calibration error.

If the value of δ falls outside of the specified tolerance limits for the UUT parameter, then it is typically deemed to be OOT. However, errors in the calibration process can result in an incorrect OOT assessment (false-reject) or incorrect in-tolerance assessment (false-accept).

The probability that the UUT parameter is in-tolerance is based on the calibration result and on its associated uncertainty. All relevant calibration errors must be identified and combined in a way that yields viable uncertainty estimates.

For illustration, the calibration results for an individual (i.e., serial numbered) load cell selected from the manufacturer/model group described in Section 2 will be evaluated. The confidence limits listed in Table 4 for this manufacturer/model group will used to assess the probability that the measured voltage output from the load cell is in-tolerance.

In this example, the load cell is calibrated using a weight standard that has a stated value of 3.02 lb$_f$ and expanded uncertainty of ± 0.01 lb$_f$. During calibration, an excitation voltage of 8 VDC ± 0.25 V is supplied to the load cell. The calibration weight is connected and removed from the load cell several times to obtain repeatability data.

The load cell output voltage is measured using a digital multimeter. The manufacturer's published accuracy and resolution specifications for the DC voltage function of the digital multimeter are listed in Table 5.

Table 5. Multimeter DC Voltage Specifications

Specification	Value	Units
200 mV Range Resolution	0.01	mV
200 mV Range Accuracy	0.05% of Reading + 2 digits	mV

The applicable calibration errors are listed below.

- Calibration weight bias, ε_W
- Excitation voltage bias, $\varepsilon_{V_{Ex}}$
- DC voltmeter digital resolution error, $\varepsilon_{V_{res}}$
- DC voltmeter bias, $\varepsilon_{V_{acc}}$
- Repeat measurements error, $\varepsilon_{V_{rep}}$

Weight Standard (*W*)
The 3.02 lb$_f$ weight standard has an expanded uncertainty of ± 0.01 lb$_f$. In this analysis, these limits are interpreted to represent a coverage factor, *k*, equal to 2. The associated error distribution is characterized by the normal distribution.

Excitation Voltage (V_{Ex})
The $\pm\, 0.25$ V excitation voltage error limits are interpreted to be 95% confidence limits for a normally distributed error.

DC Voltmeter Resolution (V_{res})
The digital display resolution is specified as 0.01 mV and the resolution error limits are $\pm\, 0.005$ mV (half the resolution). The limits are the minimum 100% containment limits for a uniformly distributed error.

DC Voltmeter Accuracy (V_{acc})
The overall accuracy of the DC voltage reading for the 0 to 200 mV range is specified to be $\pm\,(0.05\%$ of reading + 2 digits). These error limits are interpreted to be 95% confidence limits for a normally distributed error.

Repeatability (V_{rep})
The error resulting from repeat measurements can result from various physical phenomena such as temperature variation or the act of removing and re-suspending the calibration weight multiple times. Uncertainty due to repeatability error is estimated from the standard deviation of the measurement data listed in Table 6.

Table 6. DC Voltage Readings

Repeat Measurement	Measured Output Voltage (mV)	Offset from Nominal Output[10] (mV)
1	9.85	0.19
2	9.80	0.14
3	9.82	0.16
4	9.84	0.18
5	9.80	0.14
Average	9.82	0.16
Std. Dev.	0.023	0.023

The load cell calibration equation is given in equation (11).

$$LC_{cal} = W \times S \times V_{Ex} + V_{res} + V_{acc} + V_{rep} \tag{11}$$

Nominal values and error limits for the parameters used in the load cell calibration output equation are listed in Table 7. The normal distribution is applied for all parameters except V_{res}, which has a uniform distribution.

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[10] In the analysis of the calibration results, the nominal load cell output = 3.02 lb$_f$ $\times$ 0.4 mV/V/lb$_f$ $\times$ 8 V = 9.66 mV.

Table 7. Parameters used in Load Cell Calibration Equation

Param. Name	Description	Nominal or Average Value	Error Limits	Confidence Level
W	Applied Load	3.02 lb$_f$	$\pm$ 0.01 lb$_f$	95.45
S	Load Cell Sensitivity	0.4 mV/V/lb$_f$		
V_{Ex}	Excitation Voltage	8 V	$\pm$ 0.250 V	95
V_{res}	Voltmeter Resolution	0 mV	$\pm$ 0.005 mV	100
V_{acc}	Voltmeter Accuracy	0 mV	$\pm$ (0.05% Rdg + 0.02mV)	95
V_{rep}	Repeatability	0.22 mV		

The error model for the load cell calibration is given in equation (12).

$$\varepsilon_{cal} = c_W \varepsilon_W + c_{V_{Ex}} \varepsilon_{V_{Ex}} + c_{V_{res}} \varepsilon_{V_{res}} + c_{V_{acc}} \varepsilon_{V_{acc}} + c_{V_{rep}} \varepsilon_{V_{rep}} \tag{12}$$

Applying the variance operator to equation (12) and noting that there are no correlations between errors, gives

$$\begin{aligned} \mathrm{var}\left(\varepsilon_{cal}\right) = {}& c_W^2 \ \mathrm{var}\left(\varepsilon_W\right) + c_{V_{Ex}}^2 \ \mathrm{var}\left(\varepsilon_{V_{Ex}}\right) + c_{V_{res}}^2 \ \mathrm{var}\left(\varepsilon_{V_{res}}\right) \\ & + c_{V_{acc}}^2 \ \mathrm{var}\left(\varepsilon_{V_{acc}}\right) + c_{V_{rep}}^2 \ \mathrm{var}\left(\varepsilon_{V_{rep}}\right) \end{aligned} \tag{13}$$

Therefore, the uncertainty in the load cell calibration can be expressed as

$$u_{cal} = \sqrt{ c_W^2 u_W^2 + c_{V_{Ex}}^2 u_{V_{Ex}}^2 + c_{V_{res}}^2 u_{V_{res}}^2 + c_{V_{acc}}^2 u_{V_{acc}}^2 + c_{V_{rep}}^2 u_{V_{rep}}^2 } \tag{14}$$

The uncertainty in the load cell output is computed from the uncertainty estimates and sensitivity coefficients for each parameter. The partial derivative equations used to compute the sensitivity coefficients are listed below.

$$\begin{aligned} c_W &= \frac{\partial LC_{out}}{\partial W} = S \times V_{Ex} = 0.4 \ \text{mV/V/lb}_f \times 8 \ \text{V} \\ &= 3.2 \ \text{mV/lb}_f \end{aligned}$$

$$\begin{aligned} c_{V_{Ex}} &= \frac{\partial LC_{out}}{\partial V_{Ex}} = W \times S = 3.02 \ \text{lb}_f \times 0.4 \ \text{mV/V/lb}_f \\ &= 1.21 \ \text{mV/V} \end{aligned}$$

$$c_{V_{res}} = \frac{\partial LC_{out}}{\partial V_{res}} = 1 \qquad c_{V_{acc}} = \frac{\partial LC_{out}}{\partial V_{acc}} = 1 \qquad c_{V_{rep}} = \frac{\partial LC_{out}}{\partial V_{rep}} = 1$$

The weight standard bias uncertainty is estimated to be

$$u_W = \frac{0.01 \text{ lb}_f}{\Phi^{-1}\left(\frac{1+0.9545}{2}\right)} = \frac{0.01 \text{ lb}_f}{2} = 0.005 \text{ lb}_f.$$

The excitation voltage bias uncertainty is estimated to be

$$u_{V_{Ex}} = \frac{0.25 \text{ V}}{\Phi^{-1}\left(\frac{1+0.95}{2}\right)} = \frac{0.25 \text{ V}}{1.9600} = 0.1276 \text{ V}.$$

The voltmeter digital resolution uncertainty is estimated to be

$$u_{V_{res}} = \frac{0.005 \text{ mV}}{\sqrt{3}} = \frac{0.005 \text{ mV}}{1.732} = 0.0029 \text{ mV}.$$

The voltmeter bias uncertainty is estimated to be

$$u_{V_{acc}} = \frac{\left(9.658 \text{ mV} \times \frac{0.05}{100} + 0.02 \text{ mV}\right)}{\Phi^{-1}\left(\frac{1+0.95}{2}\right)} = \left(\frac{0.0048 \text{ mV} + 0.02 \text{ mV}}{1.9600}\right)$$

$$= \frac{0.0248 \text{ mV}}{1.9600} = 0.0127 \text{ mV}.$$

The uncertainty due to repeatability in the load cell voltage measurements is equal to the standard deviation of the sample data.

$$u_{V_{rep}} = 0.023 \text{ mV}$$

The sample mean is the quantity of interest in this analysis. Therefore, the uncertainty in the mean value should be included in the calculation of the uncertainty in the load cell output voltage. The uncertainty in the mean value is defined as

$$u_{\overline{V}_{rep}} = \frac{u_{V_{rep}}}{\sqrt{n}} \tag{15}$$

where *n* is the sample size. The uncertainty in the mean value is estimated to be

$$u_{\overline{V}_{rep}} = \frac{0.023 \text{ mV}}{\sqrt{5}} = \frac{0.023 \text{ mV}}{2.236} = 0.0103 \text{ mV}.$$

The estimated uncertainties and sensitivity coefficients for each parameter are summarized in

Table 8.

Table 8. Estimated Uncertainties for Load Cell Calibration

Param. Name	Nominal or Stated Value	± Error Limits	Conf. Level	Standard Uncertainty	Sensitivity Coefficient	Component Uncertainty
W	3.02 lb$_f$	± 0.01 lb$_f$	99	0.005 lb$_f$	3.2 mV/lb$_f$	0.0160 mV
V_{Ex}	8 V	± 0.25 V	95	0.1276 V	1.21 mV/V	0.1544 mV
V_{acc}	0 mV	± 0.0248 mV	95	0.0127 mV	1	0.0127 mV
V_{res}	0 mV	± 0.005 mV	100	0.0029 mV	1	0.0029 mV
$u_{\overline{V}_{rep}}$	0.16 mV			0.0103 mV	1	0.0103 mV

The total uncertainty in the load cell calibration output is computed by taking the root sum square of the component uncertainties.

$$u_{cal} = \sqrt{(0.0160\,\text{mV})^2 + (0.1544\,\text{mV})^2 + (0.0127\,\text{mV})^2 + (0.0029\,\text{mV})^2 + (0.0103\,\text{mV})^2}$$
$$= \sqrt{0.0244\,\text{mV}^2} = 0.156\,\text{mV}$$

The pareto chart, shown in Figure 2, indicates that excitation voltage is the largest contributor to the overall uncertainty. The uncertainties due to the weight standard, voltmeter and repeatability provide much lower contributions to the overall uncertainty.
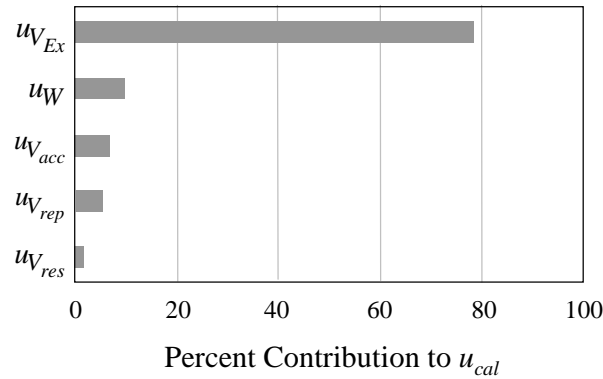

Percent Contribution to u_{cal}

Figure 2 Pareto Chart for Load Cell Calibration Uncertainty

The Welch-Satterthwaite formula is used to compute the degrees of freedom for the uncertainty in the load cell output voltage.

$$\nu_{u_{cal}} = \frac{u_{cal}^4}{\dfrac{u_W^4}{\infty} + \dfrac{u_{V_{Ex}}^4}{\infty} + \dfrac{u_{V_{acc}}^4}{\infty} + \dfrac{u_{V_{res}}^4}{\infty} + \dfrac{u_{\overline{V}_{rep}}^4}{4}} = 4 \times \frac{u_{cal}^4}{u_{\overline{V}_{rep}}^4} \tag{16}$$

The degrees of freedom for u_{cal} is computed to be

$$v_{u_{cal}} = 4 \times \frac{(0.156)^4}{(0.0103)^4} = 4 \times 52619.84 = 210,479.3 \cong \infty$$

The difference between the average measured load cell voltage output and the nominal or expected output for an applied load of 3.02 lb_f is

$$\overline{\delta} = (9.82 - 9.66)\,\text{mV} = 0.16\,\text{mV}$$

where $\overline{\delta}$ is an estimate of the bias in the load cell output, $e_{UUT,b}$, at the time of calibration. The confidence limits for the load cell bias can be expressed as

$$\overline{\delta} \pm t_{\alpha/2,\nu} \times u_{cal} \tag{17}$$

For a 95% confidence level, $t_{0.025,\infty} = 1.9600$ and the confidence limits for $e_{UUT,b}$ are computed to be

$$0.16\,\text{mV} \pm 1.96 \times 0.156\,\text{mV} \quad \text{or} \quad 0.16\,\text{mV} \pm 0.31\,\text{mV}.$$

4.1 In-tolerance Probability

Figure 3 shows the $\varepsilon_{UUT,b}$ probability distribution for the population of manufacturer/model load cells. The spread of the distribution is based on the specification tolerance limits computed in Section 3 for a 3 lb_f applied load. The calibration result, $\overline{\delta}$, provides an estimate of the unknown value of $\varepsilon_{UUT,b}$ for the individual load cell.
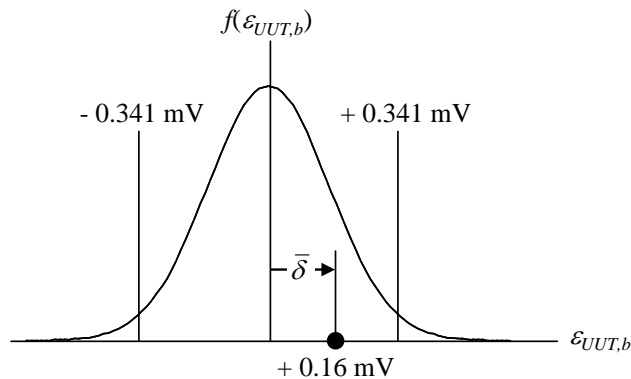

Figure 3 Load Cell Bias Distribution – 3 lb_f Input Load

Given the value of $\overline{\delta} = 0.16\,\text{mV}$ observed during calibration, it appears that the load cell output is in-tolerance. However, in deciding whether the load cell output is in-tolerance or not, it is important to consider that $\overline{\delta}$ is also affected by the bias in the calibration weight, ε_W, the bias in the excitation voltage, $\varepsilon_{V_{Ex}}$, and the bias in the digital voltmeter reading, $\varepsilon_{V_{acc}}$. Consequently, the actual bias in the load cell output voltage may be larger or smaller than 0.16 mV.

While the value of $\varepsilon_{UUT,b}$ for the calibrated load cell is unknown, there is a 95% confidence that it is contained within the limits of $0.16\,\text{mV} \pm 0.31\,\text{mV}$. Figure 4 shows the probability distribution for $\varepsilon_{UUT,b}$ centered around $\bar{\delta} = 0.16\,\text{mV}$. The black bar depicts the $\pm\,0.31$ mV confidence limits and the shaded area depicts the probability that $\varepsilon_{UUT,b}$ falls outside of the manufacturer specification limits.
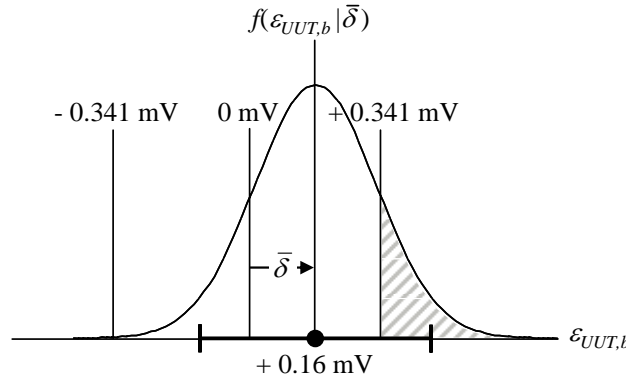

Figure 4 OOT Probability of Calibrated Load Cell

While the OOT probability is lower than the in-tolerance probability, it may introduce a significant risk of falsely accepting a non-conforming item or parameter. Bayesian analysis methods are employed to estimate UUT biases and compute in-tolerance probabilities based on *a priori* knowledge and on measurement results obtained during testing or calibration [6].

Prior to calibration, the uncertainty in the UUT bias, $u_{UUT,b}$, is estimated from the probability distribution for $\varepsilon_{UUT,b}$, the specification tolerance limits and the associated confidence level (*a priori* in-tolerance probability). First, the in-tolerance probability prior to calibration is computed by integrating the distribution function, $f(\varepsilon_{UUT,b})$, between the $\pm\,L$ tolerance limits

$$P(in) = \int_{-L}^{L} f(e_{UUT,b})\,de_{UUT,b} = 2\Phi\left(\frac{L}{u_{UUT,b}}\right) - 1 \tag{18}$$

where Φ is the normal distribution function. Rearranging equation (18), the pre-calibration estimate of the UUT bias uncertainty is obtained from

$$u_{UUT,b} = \frac{L}{\Phi^{-1}\left(\dfrac{1 + P(in)}{2}\right)} \tag{19}$$

where Φ^{-1} is the inverse normal distribution function.

After calibration, the measurement results $\bar{\delta}$ and u_{cal} are used to estimate the parameter bias of the individual UUT.

$$\beta = \frac{u_{UUT,b}^2}{u_A^2}\,\bar{\delta} \tag{20}$$

where

$$u_A = \sqrt{u_{UUT,b}^2 + u_{cal}^2}\ . \tag{21}$$

From equations (20) and (21), it can be seen that the Bayesian estimate of the UUT parameter bias will be less than or equal to calibration result, $\bar{\delta}$. For example, if the values of $u_{UUT,b}$ and u_{cal} are equal, then $\beta = \bar{\delta}/2$. Conversely if u_{cal} is much smaller than $u_{UUT,b}$, then $\beta \cong \bar{\delta}$.

The uncertainty in the parameter bias is estimated to be

$$u_\beta = \frac{u_{UUT,b}}{u_A}\,u_{cal}\ . \tag{22}$$

Finally, the post-calibration in-tolerance probability for the UUT parameter is computed from equation (23).

$$P(in) = \Phi\left(\frac{L+\beta}{u_\beta}\right) + \Phi\left(\frac{L-\beta}{u_\beta}\right) - 1 \tag{23}$$

The 95% confidence limits for a given manufacturer/model load cell were computed and summarized in Table 4. These limits were computed using uncertainty analysis methods to combine the manufacturer specifications for different applied loads. From equation (19), the uncertainty in the load cell bias prior to calibration is estimated to be

$$u_{UUT,b} = \frac{0.341\ \text{mV}}{\Phi^{-1}\left(\dfrac{1+0.95}{2}\right)} = \frac{0.341\ \text{mV}}{1.9600} = 0.174\ \text{mV}.$$

This bias uncertainty is equivalent to the standard deviation of the probability distribution for the population of manufacturer/model load cells, shown in Figure 3.

The calibration results of an individual load cell, for a 3 lb$_f$ applied load, were also analyzed. Using the calibration results of $\bar{\delta} = 0.16\ \text{mV}$ and $u_{cal} = 0.156\ \text{mV}$, the bias in the load cell output is computed from equations (20) and (21).

$$\beta = \frac{(0.174\ \text{mV})^2}{(0.174\ \text{mV})^2 + (0.156\ \text{mV})^2} \times 0.16\ \text{mV} = \frac{0.0303\ \text{mV}^2}{0.0546\ \text{mV}^2} \times 0.16\ \text{mV}$$

$$= 0.554 \times 0.16\ \text{mV}$$

$$= 0.09\ \text{mV}$$

The uncertainty in this bias is computed from equation (22).

$$u_\beta = \frac{0.174 \text{ mV}}{0.234 \text{ mV}} \times 0.156 \text{ mV} = 0.746 \times 0.156 \text{ mV}$$
$$= 0.116 \text{ mV}$$

Finally, equation (23) is used to compute the probability that the load cell output is in-tolerance during calibration.

$$P(in) = \Phi\left(\frac{0.341 \text{ mV} + 0.09 \text{ mV}}{0.116 \text{ mV}}\right) + \Phi\left(\frac{0.341 \text{ mV} - 0.09 \text{ mV}}{0.116 \text{ mV}}\right) - 1$$
$$= \Phi\left(\frac{0.431}{0.116}\right) + \Phi\left(\frac{0.251}{0.116}\right) - 1$$
$$= \Phi(3.716) + \Phi(2.164) - 1$$
$$= 0.9999 + 0.9848 - 1$$
$$= 0.985 \text{ or } 98.5\%.$$

The risk of falsely accepting the load cell output as in-tolerance is

$$FAR = 1 - P(in)$$
$$= 1 - 0.985$$
$$= 0.015 \text{ or } 1.5\%.$$

Applying Bayesian methods to analyze pre- and post-calibration information provides an explicit means of estimating the in-tolerance probability of MTE parameters. In most cases, the single variable with the greatest impact on measurement decision risk is the *a priori* in-tolerance probability of the UUT parameter [6]. Unfortunately, many manufacturers don't report in-tolerance probabilities or confidence levels for their MTE specifications. In such cases, a simplified relative accuracy criterion is often used to control measurement decision risk.

4.2 Relative Accuracy Criterion

Historically, the control of measurement decision risk has been embodied in requirements specifying the relative accuracy of the test or calibration process to the accuracy of the UUT parameter or characteristic being tested or calibrated [12, 13]. The common practice has been to require that the relative ratio of the accuracy of the UUT parameter to the collective accuracy of the calibration standards (expressed as uncertainty) must be at least 4 to 1 (4:1).

The 4:1 test accuracy ratio (TAR) requirement means that the specified tolerance for the UUT parameter must be greater or equal to four times the combined uncertainties of all of the reference standards used in the calibration process. The effectiveness of this risk control requirement is debatable, in part, because of the lack of agreement regarding the calculation of TAR.

A more explicit relative accuracy requirement has been defined in ANSI/NCSL Z540.3:2006. This standard states that where it is not practicable to estimate FAR, the test uncertainty ratio (TUR) shall be equal to or greater than 4:1. TUR is defined as the ratio of the span of the UUT tolerance limits to twice the 95% expanded uncertainty of the measurement process used for calibration. TUR differs from TAR in the inclusion of all pertinent measurement process errors.

$$\text{TUR} = \frac{L_1 + L_2}{2U_{95}} \tag{24}$$

where U_{95} is equal to the standard uncertainty of the measurement process, u_{cal}, multiplied by a coverage factor, k_{95}, that corresponds to a 95% confidence level.

$$U_{95} = k_{95}u_{cal} \tag{25}$$

In ANSI/NCSL Z540.3:2006, $k_{95} = 2$ and TUR is valid only in cases where the two-sided tolerance limits are symmetric (i.e., $L_1 = L_2$). Therefore, the UUT tolerance limits would be expressed in the form $\pm L$, and TUR expressed as

$$\text{TUR} = \frac{L}{U_{95}}. \tag{26}$$

While the TUR $\geq$ 4:1 requirement is intended to provide some loose control of false accept risk, it doesn't provide any information about the conformance of the UUT parameter with the specification tolerance limits. For example, the TUR for the load cell calibration example is computed to be

$$\text{TUR} = \frac{0.341 \text{ mV}}{2 \times 0.156 \text{ mV}} = \frac{0.341 \text{ mV}}{0.312 \text{ mV}} = 1.09.$$

While this TUR fails to meet the $\geq$ 4:1 requirement, the corresponding FAR calculated in Section 4.1 does meet the $\leq$ 2% requirement. In this case, TUR may provide some figure of merit about the calibration process, but it does not provide an assessment of FAR.

Since *a priori* in-tolerance probability of the UUT parameter is crucial for the evaluation of measurement decision risk, the TUR 4:1 criterion is not considered to be a true "risk control" method. So, when should the 4:1 TUR criterion be used? When absolutely no information about the *a priori* in-tolerance probability is available.

5   Calibration Interval Analysis
Periodic calibration comprises a significant cost driver in the life cycle of MTE. It also provides a major safeguard in controlling uncertainty growth and reducing the risk of substandard MTE performance during use. The goal in establishing MTE calibration intervals is to meet measurement reliability requirements in a cost-effective manner.

MTE attributes or parameters are subject to errors arising from transportation, drift with time,

use and abuse, environmental effects, and other sources. Consequently, the MTE parameter bias may increase, remain constant or decrease, but the uncertainty in this bias always increases with time since calibration.

The growth in uncertainty over time corresponds to an increased out-of-tolerance probability over time, or equivalently, to a decreased in-tolerance probability or measurement reliability over time. In most cases, mathematical reliability models are established from historical in-tolerance or out-of-tolerance data obtained from the calibrations of a family of MTE (i.e., manufacturer/model) or a group of similar MTE. Absent sufficient historical data, initial MTE calibration intervals are often based on manufacturer recommended intervals.

5.1 Reliability Modeling
The primary objective of reliability modeling is the establishment of calibration intervals that ensure that appropriate measurement reliability targets are met. A reliability model predicts the in-tolerance probability for the parameter bias population as a function of time elapsed since measurement. It can be thought of as a function that quantifies the *stability* of the population.

The measurement reliability of the parameter bias at time *t* is

$$R(t) = \int_{-L_1}^{L_2} f[\varepsilon_b(t)]d\varepsilon_b \tag{27}$$

where $f[\varepsilon_b(t)]$ is the probability density function (pdf) for the parameter bias $\varepsilon_b(t)$ and $-L_1$ and L_2 are the specification tolerance limits. For example, if $\varepsilon_b(t)$ is assumed to be normally distributed, then

$$f[\varepsilon_b(t)] = \frac{1}{\sqrt{2\pi}u(t)} e^{-[\varepsilon_b(t)-\mu(t)]/2u^2(t)} \tag{28}$$

where $u(t)$ is the bias uncertainty and $\mu(t)$ represents the expected or true parameter bias at time *t*. The relationship between L_1, L_2, $\varepsilon_b(t)$ and $\mu(t)$ is shown in Figure 5, along with the bias distribution for the MTE parameter of interest. The measurement reliability $R(t)$ is equal to the in-tolerance probability.
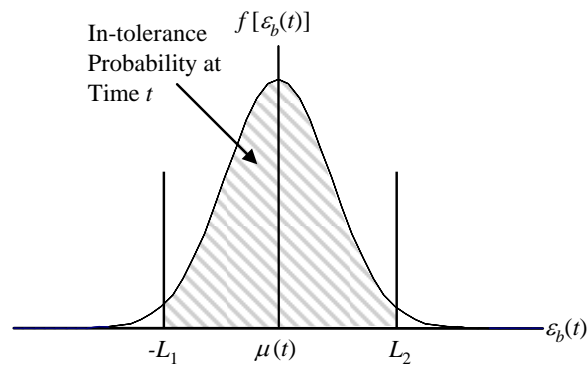

Figure 5 Parameter Bias Distribution.

The reliability model is defined by a mathematical equation that describes how measurement reliability changes over time. A calibration interval analysis application can be used to determine the reliability model that "best fits" the calibration history data and to compute the corresponding model coefficients.

If a reliability modeling application is not available, then an applicable reliability model must be chosen based on knowledge about the stability of the MTE parameter over time. Descriptions of commonly used reliability models can be found in NCLSI RP-1 *Establishment and Adjustment of Calibration Intervals*.

Once the reliability model has been established it can be used to identify the appropriate calibration interval for a desired reliability target, as shown in Figure 6. A measurement reliability target is determined by the requirements for calibration accuracy and is usually referenced to the end of the calibration interval or to a value averaged over the duration of the calibration interval.
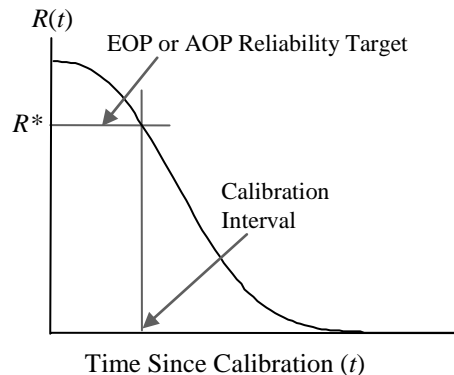

Figure 6 Measurement Reliability versus Time.

The establishment of end-of-period (EOP) or average-over-period (AOP) reliability targets involves a consideration of several trade-offs between the desirability of controlling measurement uncertainty growth and the cost associated with maintaining such control.

In practice, many organizations have found it expedient to manage measurement reliability at the instrument rather than the parameter level. In these cases, an item of MTE is considered out-of-tolerance if one or more of its parameters is found to be out-of-tolerance.

5.2 Manufacturer Specified Intervals
Calibration intervals specified by MTE manufacturers are often developed from the analysis of stability data at the parameter level. The following information is needed to implement the specified calibration interval:

- The parameter tolerance limits.

- The period of time over which the parameter values will be contained within the tolerance limits.

- The probability that the MTE parameter will be contained within the tolerance limits for the specified period of time.

Unfortunately, manufacturers may not communicate all of the necessary information to adequately adopt their interval recommendations. In this case, supporting calibration data from similar equipment, engineering design analysis and manufacturer expertise can be helpful in establishing initial calibration intervals.

6   Conclusions

As illustrated in this paper, MTE specifications are an important element of measurement quality assurance. The majority of MTE documents fall short of providing crucial information about the confidence levels associated with reported specification limits. MTE manufacturers also don't indicate the applicable probability distribution for a particular performance characteristic.

Consequently, it is difficult to apply MTE specifications without gaining further clarification or making some underlying assumptions. It is a good practice to

1. Review the specifications and highlight the MTE characteristics that need clarification.
2. Check the operating manual and associated technical documents for other useful details.
3. Request additional information and clarification from the manufacturer's technical department.

Ultimately, the MTE user must determine which specifications are relevant to their application. Therefore, a basic understanding of the fundamental operating principles of the MTE is an important requirement for proper interpretation and application of performance specifications.

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